THE MANAGEMENT OF MARKET MICROSTRUCTURE: APPLICATIONS OF OPERATIONS RESEARCH MODELS

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ABSTRACT

This paper is concern with the Management of Market structure: Application of Operation Research Models with respect to the Nigeria Capital Market. It looks at each Model from the inventory models through the information-based and game – theoretical paradigms. It looks at how models work, its strength and weaknesses. The models are examined with respect to the impact of various trading mechanisms on price formation, information findings, buoyancy and trading behaviour. This paper discusses the bid-ask spread which is of course, a major component of price; and it is a chain of many factors which includes the market maker or dealer's costs, and information availability, as well as signaling effects. It concludes that the Nigeria Capital Market will benefit from the use of Operations Research Models. It recommends that Market Operators should be educated with these Operations Research Models.

Keywords: Capital Markets, Market structure, Operations Research Models, Microstructure.

Introduction:

Operations Research Models view the trading process as a matching problem in which the market maker must use prices to balance supply and demand across time. A key factor in the inventory model is that the market maker's is concerned with inventory position; the key factor is the market maker's inventory position. While the other approaches views trading process as a game which involve traders with asymmetric information regarding the assets true value.

The models that provide the general framework in Market Microstructure are the inventory-based and information-based paradigms. Both models provide the general framework for this paper. This model constitutes the seat for Market Microstructure.

This paper is concern with the behaviour of market prices with respect to the inventory models, which is interested in the theoretical analyses of the Security Market Microstructure; the information-based models, which explains price behaviour; it allow for examination of market dynamics and hence provide insights into the adjustment process of prices (O' Hara 1995). The strategic trader models for informed and uninformed trader's addresses the difficult issue of computing the exact return to information because sequential approach is not of help. Its approach is on batch framework. In this model, the payoff to strategic behaviour can be computed because traders are cleared at a single price. With this case in hand, information found in single trades or in the bid-ask spread is deleted.

Definition of Market Microstructure:

We have varied definitions of Market Microstructure. But all is centred round finance and its application to the exchange of assets.

From Wikipedia (2008), the free encyclopedia, Market Microstructure is a branch of finance that is concerned with the details of how exchange occurs in markets. Its theory is applicable to the exchange of real or financial asset; more evidence is available on the microstructure of financial markets due to the availability of transactions data from financial markets.

According to O'Hara (1995), Market Microstructure is defined as "the study of the process and outcomes of exchanging assets under a specific set of rules".

The National Bureau of Economic Research sees Market Microstructure that is devoted to theoretical, empirical, and experimental research on the economics of Securities Markets, including the role of information in the price discovery process, the definition, measurement, control, and determinants of liquidity and transaction costs, and their implications for the efficiency, welfare, and regulation of alternative trading mechanisms and market structures.

Osamwonyi I. O. (2007) defined Market Microstructure as "an area of finance that studies the process of price formation in the securities market. It deals with how the process and structure of financial market affect price, liquidity; and trading behaviour.

This paper defines it as a subset of finance that is mainly concerned with the determination of market price of securities.

Nature of Markets and Prices:

It is interesting to know that former works on Market Microstructure looks at issues relating to the stochastic nature of supply and demand, but now it looks more on the information – aggregation properties of prices and markets. Market Microstructure now has to do with the collection of models, ranging from the inventory models, information-based models and strategic trader models. And this is the focus of this seminar paper. The rest part of paper will deal with this model as it relates to price formation in the market with respect to price mechanism.

We raised some questions and provide an attempt in answering them in this paper. How are prices set in the market? We have the traditional view but it fails to include any role for explicit trading mechanisms. We have the types of market that are used to trade financial assets. The types of markets used will serve as an introduction to analyse the models in this paper, which deals with various modelling approaches cum theory used to explain the evolution of prices in markets.

The question now is how is price determined? From standard economics paradigm; it is the intersection of supply and demand curves or where both schedules are equal for a product i.e. the equilibrium. The next questions is, how exact do we attain this equilibrium? And another question is what variable coordinates the desires of demanders and suppliers so that price emerges and trade occurs? These critical questions inform the paper either because economics do not provide enough answers. Thus, the models in this paper will be of help in answeringthese questions with particular reference to the works of researchers in this area.

The mechanics of price formation has two traditional approaches, and these are:

- To argue for its irrelevance 'cum' how market clearing was achieved was not of interest. And the behaviour as to out of equilibrium is not considered; because it is difficult to reconcile, not to talk of characterizing what its properties might be. The approaches are of two assets – (1) simplicity and (2) generality. It is assumed that trading mechanism plays no role in affecting the resulting equilibrium. This assumption is troublesome, because traders have differential information.
- The second approach is the WALRASIAN AUCTIONEER (WA). In this approach, the mechanics of price setting is the fiction of a Walrasian Auctioneer. Price formation process could be captured by the general representation of Walrasian Auctioneer. Walrasian auctioneer is simply the aggregation of traders' demands and supplies to find a market-clearing price. How the trading mechanisms did works? The trading mechanisms works with each trader providing its own demand or demand schedule, potential price is suggested, decisions are made until it gets to equilibrium. It is interesting to note here that market prices are a series of preliminary actions, and once equilibrium is attained no trading is done outside it and no recontract. In this approach trading position is not taken, it only redirects quantities from sellers to buyers. Its activity is costless, thus, no friction in exchange process. It looks at natural processes to form price. The posing question here is, is it always the case? Are there other variables attached to price formation? Note that it is not all markers that agree with Walrasian Auctioneer. The issue of trade behaviour needs to be considered. Trading mechanism has importance of its own in price formation.

The work of Demsetz (1968) is considered in the determination of prices in Securities Markets. Demsetz analyses on how the time dimension of supply and demand affects market prices. He focuses on the nature of transaction costs. It involves some costs, which could be explicit or implicit. The implicit cost here is the price of immediacy; that trading had a time dimension. Price of immediacy is paid; when there is imbalance of trade and it is impossible to find a market-clearing price, because it is not at all times that buyers equal sellers. He argues that there are two sources of supply and demand in the market. Thus, there will be two prices and not even one will characterise the equilibrium. And this will give us double demand and supply curves reflecting or showing two time frames of the trading process.

This paper looked at some general features of exchange markets and the alternative ways that price-settings rules are structured in actual markets. Many ways are involved in the the process of exchange and these are:

- Buyers and Sellers can meet directly
- Traders gather at a central setting or
- Traders use computer communicator
- Single intermediary can arrange every trade
- Individuals meets to set prices

We have explicit or implicit rules that govern the trading mechanism, no matter the setting. And the rules stand in formation and evolution of market price. Walrasian auctioneer is one of the set of rules as above. It follows a sequential process, note that no trade occur out of equilibrium. Trading volume is not available, only the amount is available (O'Callaghan 1993). Trading Mechanism can be seen as a type of trading game in which the players meet (perhaps not face to face) at some volume and act according to some rules. The players are customers, brokers, dealers who are customers too, dealers (who trade for their own), and specialists or market maters.

Paper Objectives:

The primary objectives of this paper are to contribute to knowledge and to
aid the Nigeria Capital Market in the formation of prices using operations research models, for the orderly socio-economic development of the economy; especially in the management of bid-ask spread in the hands of the dealers, or Market Makers or brokers etc. The global objective is to offer optimal operations research models for the Nigerian Capital Market.

Significance of Paper:

This paper is prepared to do basically two things:
1) to contribute to knowledge by going a step forward to evaluate the Nigerian Capital Market from the perspectives of bid-ask spread, and
2) to assist the regulatory authority the Nigerian Stock Exchange (NSE) and Securities and Exchange Commission (SEC) in terms of determining how best to form price for securities and the models to use.

Applications of Operation Research Model:
Inventory Models:

Inventory models are interested in the theoretical analysis of the Security Market Microstructure. We are looking at the security prices in the Security Market. Of old securities prices falls in the area of Macroeconomics, because all former researches on securities prices points to that. Demsetz sees it from another angle, the nature of bid and ask prices, which is focused on micro foundations of security markets. He sees security prices from the angle of Microeconomics; that is an optimising behaviour by economic agents.
The theoretical analysis of the Security Market Microstructure started with Garman (1976), they focused on the understanding how market prices come about given the nature of order flow and market – cleaning protocol. We have three research paradigms associated with these problems:
1) **Garman**: He looks on the nature of order flow in stating security prices.
2) **Stoll and Ho**: They looked on the dealer's optimization problem.
3) **Cohen, Maier, Schwartz, and Whitcomb**: Looks on the effect of multiple providers of liquidity.

The inventory model plays the role in price formation by using the bid and asks prices quotes to manage the in balance in buy and sell orders, so as to have effective inventory control. A market maker that is faced with unbalance risk uses the bid-ask prices to balance supply and demand. According to Calamia, 1999, it follows that bid-ask spread increases with the market maker's or dealer's risk aversion, the size of the transaction; the risk of asset and the time horizon. And for Amihud and Mendelson, 1980, it could simply be a reflection of the market maker or dealer's market power. The spread of bid-ask between market maker buying and selling prices may be caused by market failure, market power, dealer risk aversion, and even gravitational pull. O'Hara, 1995 opines that in the long-run, the adjustment process brings prices and inventory to actual level they would attain when order flows are balanced.

Order Arrival And Market Making 'Cum' Ruin Problem:

In this part, we are looking at how a risk neutral market maker deals with Complex Uncertainty brought by stochastic supply and demand. At equilibrium price, the quantity demanded is equal to the quantity supply. This position holds in the market outcomes.
We are looking at Garman's Model, where there is a single, monopolistic Market Maker that sets prices, receives all orders and clear trades. The dealer has objectives that are subject to constraint. In this model, the Market Maker has only one decision to set asks and bid prices. Let ask price be Pa and bid price be Pb. The ask price, Pa is that at which the Market Maker will fill orders wishing to buy the stock, while the bid price, Pb is that at which he will fill orders wishing to sell the stock. The model assures that orders are for one unit of stock. And at the beginning of time, the dealer select the bid-ask price once despite is infinite horizon.

Order arrival follows a poison process, the buy and sell orders are assumed to be poison and the time (t) waiting time between arrivals of buy (sell) orders is exponentially distributed. Thus queueing theory is the story of order arrivals. If (t) is the time of the last buy order, then the probability of a buy order arrival in the interval [t, t + Δt] is approximately λaΔt for small Δt. O' Hara, 1995, is of the view that potential in-balance is the Crux of dealer's problem. Thus, the main problem of the dealer in this model is "Staying alive".

In the period time zero (o), the market maker holds Ic (o) units of cash and Is (o) units of stock. If we allow Ic (t) and Is (t) be the units of cash and stock at time t. And Na (t) is the cumulative numbers of shares sold to traders up to time t – called the executed buy orders, then Nb (t) – called the executed sell orders; is the cumulative number of shares bought from traders at time (t). Thus, the inventory model is ruled by:

$$Ic\ (t) = Ic\ (o) + Pa\ Na\ (t) - Pb\ Nb\ (t) \quad \ldots\ldots\ldots (1) \quad \text{and}$$

$$Is\ (t) = Is\ (o) + Nb\ (t) - Na\ (t) \quad \ldots\ldots\ldots\ldots\ldots (2)$$

If we define Qk (t) to be theprobability that Ic (t) = k and Rk (t) to be the probability that Is (t) = k.

The assumption raised by Garman is that a unit of cash (say a naira) arrives with rate λaPa and departs at rate λbPb. And with the arrival and departure, we have the following probability:

- that a dealer has exactly K units of cash at time t is the sum of the following probabilities, i.e.

$$\begin{aligned} Qk\ (t) = {} & Qk - t\ (t-\Delta t)\ [\lambda a\ (Pa)\ Pa\ \Delta t]\ [1- \lambda b\ (Pb)\ Pb\ \Delta t] \\ & + Qk +1\ (t-\Delta t)\ [\lambda b(Pb)\ Pb\ \Delta t]\ [1- \lambda a(Pa)\ Pa\ \Delta t] \\ & + Qk(t-\Delta t)\ [1 - \lambda a(Pa)\ Pa\ \Delta t]\ [1- \lambda b(Pb)\ Pb\ \Delta t] \ldots\ldots\ldots\ldots (3) \end{aligned}$$

Thus, the time derivative of the probability Qk (t), we have the limit Δto of $\longrightarrow$

$$\frac{\partial\ Qk\ (t)}{\partial t} = Qk - 1(t)\ [\lambda a(Pa)\ Pa] + Qk + 1(t)\ [\lambda b(Pb)\ Pb] - Qk\ (t)\ [\lambda a(Pa)\ Pa] + \lambda b(Pb)\ Pb] \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equation (4) expresses the dynamics of the Market Maker's cash position. We have dual ruin problems in this model, they are:

- The Gambler, and
- The Market Maker's.

The gamblers failure probability is given by model/expression:

$$\left(\frac{\text{odds of losing x amount of loss}}{\text{odds of winning x amount of gain}} \quad \begin{matrix}\text{initial wealth} \\ \text{position}\end{matrix} \right)$$

If P stands for the probability someone sells to the dealer i.e. dealer "gain" a unit of stock and q stands for the probability someone buys from him i.e. loses a unit of stock; thus, P > q and dealer initial stock is So units. The probability of failure at times is given by the probability of failing at time t given that the dealer currently has S units as Pr [F/S]. He may have chance that someone may take away one unit (S – 1) units, and chance that he will get a unit of stock (S + 1) units. Then, the probability will be: Pr [F/S] = qPr [F/S-1] + qPr [F/S +1] .

Thus, the general expected failure probability is

$$Pr\ [F/So] = \left(\frac{q}{P} \quad So \right)$$

The approximate probability failure from running out of cash in continuous time context is:

$$\left(\begin{matrix} \lim\limits_{t\to\alpha} Qo\ (t) \dfrac{\sim\lambda b\ (Pb)\ Pb}{\lambda a(Pa)\ Pa} \quad Ic\ (o)/\ P \\ = 1 \quad \text{If } \lambda a\ (Pa)\ Pa > \lambda b\ (Pb)\ Pb \end{matrix} \right)$$

Note that P is mean price of the bid and ask prices. And this gives a corresponding stock failure probability of

$$\left(\begin{matrix} \lim\limits_{t\to\alpha} Ro\ (t) \dfrac{\sim\lambda a\ (Pa)}{\lambda b\ (Pb)} \quad Is\ (o) \\ = 1 \quad \text{If } \lambda a\ (Pa) > \lambda b\ (Pb) \end{matrix} \right)$$

NB: λa (Pa) is the probability of stock going out
λb (Pb) is the probability of stock in
Is (o) is the Initial stock holdings.

In all ruin problems, the dealer's failure probabilities are always positive. The dealer will always fail no matter the price he sets except the Market Maker set both the Pa and Pb at the same time. And it is only satisfy at Pa λa (Pa) > Pb λa (Pb) and λb (Pb) > λa (Pa), if and only if, it is possible.

The limitation of Garman's model is that it only allows dealers to get prices at the initial stage of trading. Thus, Amihud and Mendelson (1980), came out with a better approach to this problem, which consider how the dealer's prices change with varied inventory position with respect to time. Thus, dealer's position is viewed as a semi-marker process, such that inventory is the state variable.

The Dealer's Problem:

The dealer is the provider of immediacy; as such he needs to be compensated for the services rendered. While still sees the dealer as risk averse, Garman and Amihud– Mendelson sees the dealer as risk neutral. Dealer faces three sources of cost while providing services and these are (1) the holding cost, (2) order – processing cost and (3) cost from trading with individual.

At time 1, the dealer decision problem is to set prices for transaction to buy or sell asset, and at time 2 having liquidation of asset. The dealer borrows at risk-free rate Pf to finance his inventory and also lead if he has excess funds at Rf – risk free rate. The Market Maker risk of bankruptcy is zero because of short time period and unlimited borrowing ability.

If Wo is the dealer's initial wealth, Qp is the true value, and given that the dealer known's the "true" value of the stock, then Qi will denote the true value of a transaction in stock i, where the true value is the true price times the number of share, a positive (negative) number indication a buy (sell). Thus, the dealer's terminal wealth is given by

$$\hat{W} = Wo\,(1+\acute{R}^*) + (1+\acute{R}i)\,Qi - (1+Rf)\,(Qi - C1)$$

Where Ŕ* is the rate of return on his initial portfolio.
Ŕi is the rate of return on stock i.
Qi – C1is the cost of carrying the inventory or in the case of sale, the return on the proceeds.
Ci is the present dollar cost to the dealer of trading the amount Qi.
e.g. A dealer that buys a share Qi of value, only need to borrow Qi – C i to finance the purchase.

The dealer requires E [U (Wo (1 + Ŕ*)] = E [U(Ŵ)] because he is assume to transact if his expected utility will be unchanged.

Using Taylor series expension, and dropping terms of order higher than two, with Rf = 0, we have:

$$\frac{C\,i}{Qi} = C\,i\frac{Z}{Wo}бip\,Qp + ½\,\frac{Z}{Wo}\,бi^2\,Qi$$

if Z is the dealer's co-efficient of relative risk aversion .
Qp is the "true"naira value of stocks held in the dealer's trading account (his total inventory).
бip is the correlation between the rate of return on stock I and the rate of return on the optimal efficient portfolio.
бi ² is the variance of stock i's return.

Thus, from the immediate expression above, Ci (Qi) =C i/Qi determines the percentage naira cost that is necessary for the dealer to be willing to take that position Q stock i.

The dealer cost of providing immediacy is compensated through its trading prices. His bid and ask price does that. If we express these costs in percentage terms relative to the true price Pi*, then the optimal bid price, Pb; for a transaction with true value Qib is then

(Pi* - Pb)/ Pi* = C i (Qib).

For Qib is the "true value" of a sale to the dealer. The optimal ask price can be obtaining same way. Thus, the resultant spread will be:

$(Pa - Pb)/ Pi^* = C\, i\, (Qib) - C\, i\, (Qi^a) = [Z/Wo]\, б i^2/QI$
For $/Qi^a/ = /Qib/ = /Q/$.
In this model, inventory matters largely because dealer cannot hedge his inventory exposure.

Inter temporal Role of Inventory:

The model used here is the finite horizon model. The model used is that of Ho and Stoll (1981), which is an extension of Stoll (1978), its analysis to make a multiperiod framework; where order flow and portfolio returns are stochastic. It is similar to Garman (1976), where buy and sell orders are made by stochastic processes, and it is the dealer's pricing strategy that determines order arrival rates. The model assumes that a monopolistic dealer maximizes expected utility of terminal wealth and how dealer's attitude towards risk will affect the solution. This is, different from risk neutral intertemporal models of Garman (1976) and Amihud and Mendelson (1980). As mentioned initially, use is made of the finite horizon model (T period) dynamic programming approach to characterize the dealer's optimal pricing policy. The dealer's optimal pricing strategy is a function that states the bid and asks prices, Pa and Pb, with respect to variables that affect dealer's future utility. The variables are: (1) dealer's cash, (2) inventory, and (3) base wealth positions. Because it applies the finite horizon model, time period also affects dealer's choice.
The model assumes that true value of stock is at some value of P. As such, dealers price is Pa = P + a and Pb = P – b. And for convience dealer's choice variables are denoted by a and b; not specific prices. Also assumed is that transaction evolving as a stationery continuous-time jump process. It is assumed as a Poisson process (Garman). Given that arrival rates to buy orders is λa and sell orders, is λb, depends on the dealer's ask and bid prices. Thus, the probability of the next trade i.e. buys or sell is influence by the dealer's price because it is Stochastic. The dealer is assumed to face uncertainty, as such, future wealth is random. The return on portfolio follow a non standard Wiener process, thus, dealer earns some random return over time.
Where there is no transaction, Portfolio growth, dX is given by:

$$dX = X\, dt + Xdzx$$

Where ᵞx is the mean return per unit time
Zx is the Wiener process with mean Zero
$б x^2$ is the instantaneous variance.
The dealer's portfolio is made up of (1) Cash, (2) Stock and (3) Base wealth. As the dealer's buys and sells securities, its cash level changes, with any balance in the account earning risk-free rate r.Thus the value of cash a/c is
$df = rf\, dt – (P-b)\, dqb + (P+a)\, dqa$
Where qb and qa are buys and sales of securities.
The value of dealer's stock or inventory position is

$$dI = rlIdt + Pdqb - Pdqa\ Idzl$$

The features of this model with respect to inventory are:

- Valuation of inventory is at the intrinsic value of stock.
- Valuation of inventory changes dues to changes in its sizes with respect to qb and qa; and changers in its value resulting from the diffusion term Idzl and the drift term rIIdt. And it poses a technical problem because it assumes inventory to be constant at price P.

The dealer's portfolio also includes base wealth, Y, and the change in its value is given by
$dY = ry\, Ydt + Ydzy$

The dealer have to choose the bid and ask prices which will maximize its expected utility of terminal wealth [E[u(W_T)]], thus solving his pricing problem; where his wealth is

$$W_T = F_T + I_T + Y_T$$

The value function J(*) solves the bellman equation such that

$$J(t, F, I, Y) = \max_{a,b} [E [u(W_T)]/t, F, I, Y],$$

is the maximized value of the problem that gives J(*).

Where U is the Utility function

a and b are the ask and bid adjustments,

t, F, I, Y is the state variables time, cash, inventory and base wealth respectively.

Thus, the recursion relation implored by the principle of optimality is given by

dJ (t, F, I, Y) = o and J(t, F, I, Y) = U (W_T), because there is no intermediate consumption before time T.

The dealer's solution to his problem can only be solved using Ito's Lemman application, when we consider a smooth function Y= f(x, t); with a continuous time, as well applying Stochastic Calculus.

Where t is the time

x is some well – defined Ito process.

$dx = Udt + \sigma dz.$

To maximise Y, we choose x, and take the derivation of Y and it will be as follows:

$$dY = \frac{df}{dt} dt + \frac{df}{dx} dx + \frac{1}{2} \frac{d^2f}{dx^2} (dx)^2$$

$$= \frac{df}{dt}dt + \frac{df}{dx} [udt + \sigma dz] + \frac{1}{2} \frac{d^2f}{dx^2} \sigma^2 dx$$

If will collect dt terms, we now have:

$$dY = \left[\frac{df}{dt} + \frac{df}{dx} u + \frac{1}{2} \frac{d^2f}{dx^2} \sigma^2 \right] dt + \frac{df}{dx} \sigma dz.$$

The function above is Ito's Lemma and it helps to calculate the derivative of a function that depends on time and a stochastic process.

The equation dJ (t, F, I, Y) = o and J(t, F, I, Y) = U (W_T) is a dealer problem which maximises the value of J (*) function, that depends on time, and cash, stock, as well as base wealth. If we rewrite the partial differential equation implied by J(t, F, I, Y) = U (W_T) and using Ito's Lemma, we have

$$\frac{dJ}{dt} = J_t + L J + \max_{a,b} \{\lambda a \{J(t, F + pQ + aQ, I-PQ, Y) -$$

$$J(t, F, I, Y)] \} + \lambda b [J(t, F - pQ + bQ, I+PQ, Y) -$$
$$J(t, F, I, Y)] = o$$

Where Jt is the time derivative and L is the operator defined as

$$L J = JF\, r\, F + JI\, r\, II + J\, y\, r\, yY + \frac{1}{2} Jyy\, \sigma^2 yY^2$$
$$+ \frac{1}{2} J\, II\, \sigma I^2\, I^2 + J\, I\, Y\, \sigma IY\, IY.$$

In the above expression, the dealer is maximising expected utility and thus, the expectation of the dz term is zero at the optimum. It determines the solution to the dealer's problem; and requires that one solves explicitly for the J(*) function.

With respect to Ho and Stoll, they only take the first-order approximation of Taylor's series expansion of the max term in

$$J(t, F, I, Y) = \max_{a,b} [E [u(W_T)]/t, F, I, Y] –$$

Such that:

J(t, F + Q + aQ, I – Q, Y) = J(t, F + Q, I – Q, Y) + JF(t, F + Q, I – Q, Y)aQ, and similar for the bid term.

And the error of this approximation is on the order of a^2.

The symmetric linear demand and supply to the dealer is assumed also by Ho and Stoll, so that $\lambda a = \lambda(a) = \alpha - \beta a$, and $\lambda b = \lambda(b) = \alpha + \beta b$.

And sell operator, S, is defined by:

SJ =S(J (t, F, I, Y) = J(t, F + Q, I – Q, Y)

and the buy operator, β is

BJ = B [J (t, F, I, Y) = J(t, F + Q, I – Q, Y)

The two operators intend to capture incremental effects on the dealer's utility of changing his holdings by Q units. Thus, utility increases if transaction taker dealer closer to desired position and decreases if it takes dealer away from desired portfolio. With this the dealer's problem is restated as:

$$Jr = LJ + \text{Max} \{ \lambda(a)a\ QSJF - \lambda(a) [J(*) - SJ]$$
$$+ \lambda(b)b\ QBJF - \lambda(b) [J(*) - BJJ]$$

With respect to, the bid case, the first order conditions to the dealer problem can be solved with:

$$b^* = \frac{\alpha}{2\beta} + \frac{J(*) - BJ(*)}{2BQJF}$$

to obtain the dealer's optimal prices.
Where α, and β are parameters of the supply and demand functions.

The dealer's problem is to set bid and ask prices, bt and at, and it will help to solve:

$$\text{Max } E [\sum_{j=o}^{\alpha} \Phi^{j}\ u(\sum_{t=l}^{n} Ħjt]$$

Where Φ is the discount rate
j is the index for trading days,
u is a strictly concave utility function
t is the index of trading periods in each day,
πjt is the trading profit in period t of day j

The order flow of Market Maker's is made up of limit orders to buy or sell and market orders to buy or sell. Limit orders are linear functions of the price and this are placed by cumulative order functions defined as integrals of the incremental orders. Thus, the limit orders to buy from the dealer in period t,

denoted A^L is given by:

$$A_t^L = \alpha^L - a_t Y^L = ƒ^{Lā}_{at} qa(at)\ dat$$

And limit order to sell is denoted by $ß^L$

i.e. $\beta t^L = \beta^L + bt\ \Phi^L = ƒ^{bl}_{b}\ qb(bt)\ dbt$

Where L is the superscripted variables, it refers to the limit order book.
α,Y, β,and Φ are parameters of limit order flow, q functions are the incremental orders at each price, ā and ъare the limit of the integration.

While the period's market order flow that is made up of price-dependent and liquidity-based orders. Market Maker form is expectation about the market order flow using the information from his limit orders; thus, the market order flow is given by:

$$\tilde{A}t^m = \alpha^m - at\ r^m + \acute{\omega}t$$
$$\beta t^m = \beta^m + bt\ \Phi^m + \acute{\varepsilon}t$$

Where ώt and έt terms are random variable.

Now, with respect to daily settlement, this made inventory to affect current cash flow and dealer's future operation. The Market Maker's dynamic program for any trading is given by:

$$\text{Max } E [u \sum_{t=l}^{n} \pi t + V(In)]$$

Where V is the dealer's derived value function.

However, the dealer's last decision in a trading day is to get an and bn to maximise:

$$u\left(\sum_{t=l}^{n-l} \pi t + an\,(\alpha - anY + \acute{\omega}n) - (\beta + bn\,\Phi + \acute{\varepsilon}n)\right.$$
$$+ rP(In\text{-}l + \beta + bn\,\Phi + \acute{\varepsilon}n - \alpha + anY - \acute{\omega}n)\,)$$
$$+ V(In\text{-}l + \beta + bn\,\Phi + \acute{\varepsilon}n - \alpha + anY - \acute{\omega}n)$$

Subject to:

$$\alpha^{L} - anY^{L}$$

$$\beta^{L} + bnY^{L}$$

Note that dealer's profit in period n, πn is obtain in

$$an(\alpha - an\,Y + \acute{\omega}n) - bn\,(\beta + bn\Phi + \acute{\varepsilon}n) +$$
$$rp(In\text{-}l + \beta + bn\,\Phi + \acute{\varepsilon}n - \alpha + anY - \acute{\omega}n),$$

The direct cash flow effects are the first two terms and the cash flow cost of financing or lending the resulting inventory is the last term. Thus, to obtain the bid and ask prices, the first-order condition can be solved, if we assume the interior solutions of:

$$an = \frac{\alpha}{2r} + \frac{E(U+\acute{\omega}n)}{E(Ut)2r} + \frac{rE(VtP)}{2E(Vt)} + \frac{E(Vt)}{2E(Vt)}$$

$$bn = \frac{\beta}{2\Phi} - \frac{E(U+\acute{\varepsilon}n)}{E(Ut)2\Phi} + \frac{rE(UtP)}{2E(Ut)} + \frac{E(Vt)}{2E(Ut)}$$

Thus, trading prices can be solved for the spread given by:

$$an - bn = \frac{\alpha\Phi + \beta r}{2\Phi r} + \frac{\Phi E(\acute{\omega}n) + rE(\acute{\varepsilon}n)}{2\Phi r} + \frac{\Phi CoV(Ut, \acute{\omega}n) + (rCoV(U', \acute{\varepsilon}n)}{2\Phi rE(U')}$$

It is akin to that of Ho and Stoll, since it can be viewed as a risk neutral spread and adjustment for risk.

The dealer's problem can be assumed to be:

$$\text{Max } E\,[-\exp(-\bar{l}\sum_{t=l}^{n} \bar{\pi}t) - \exp(-\bar{p}In)]$$

Where C and d are parameters which stand as the Market Maker's coefficient of absolute risk aversion associated with trading profits and overnight inventory respectively.(O'Hara and Oldfield).

With respect to prices and inventories in Competitive Markets, if we assume transaction in stocks M and N to be independent, then a dealer with inventories of M naira value in stock M and N naira value in stock N has a reservation buying fee for stock M of:

$$bm = \tfrac{1}{2}\,\sigma^2 MR\,(Q + 2Im)$$

and a selling fee for stock M of

$$am = \tfrac{1}{2}\,\sigma^2 MR\,(Q + 2Im).$$

Where Q is a fixed transaction value

R is the discounted co-efficient of dealer absolute risk aversion.

$Im = M + (\sigma NM/\sigma M^2)N$ is the overall value of the dealer's inventory position; and it depends on the return variance of stock $M(\sigma m^2)$ and the co-variance of return between stocks M and N.

Information-Based Models:

The information-Based Models considers the theory of market prices and its spread, in this model inventory has no role to play. It all started with the work of Bagehot (1971). This model allows for examination of market dynamics and as such give insights into the adjustment process of prices.

According to Copeland and Galari, 1983; O'Hara, 1995; and Calamia, 1999; are of the view that the Market Maker must quote large bid-ask spread where there are informed traders, as this will compensate them for losses in trading with them. Even if the Market Maker have no costs and he is risk neutral in a competitive market; the bid-ask spread still exists.

With respect to informed and uninformed traders, a trade that comes from an informed trader is having a probability of πI and if the trade comes from uninformed trader its probability is $I - \pi I$.
And with assumption of Copeland and Galai, when an uninformed will buy its probability is (πBL) he will sell, is having a probability of πSL, and probability he will not buy or sell is πNL. And to the informed trader, he is assuming to trade so that he maximises profit. A market trader who is informed has a probability of πI, and as such makes the maker loss or expected loss to be:

$$\int_{P_A}^{\alpha}(P-PA)\, f(P)dP + \int_{o}^{P_B}(P_B-P)f(P)dP.$$

Where P_B is the dealer's bid price
P_A is the dealer's ask price
P is the "true" stock price.
And with and uninformed trader, the market makers expected gain is:

$$\pi_{BL}\,(P_A-P) + \pi_{SL}(P-P_B) + \pi_{NL}\,(o)$$

The question now is, what happens if the dealer do not know the trader, he simply add both probabilities i.e. $-\pi_I$ and $(I - \pi_I)$.

In Glosten – Milgrom Model, all trades takes place at either the market maker's bid or ask prices, and trade are involves one unit of asset, traded asset has an eventual value V. There are no transaction costs or explicit costs etc. Some traders have information about V while some do not have.
If an informed agents have knowledge of the true value of the stock, this could be high or low and are represented by V or V. Let S_l represent event, a trader wants to sell the stock to the market maker and Bl represent, he wants to buy from the market maker. Then the market maker will set bid and ask price with:

$$a_l = E\,[V/B_l] = \underline{V}\,Pr\{\bar{V} = \underline{V}/ B_l\} + VPr\,\{V=V/B_l\}$$

$$b_l = E\,[V/S_l] = \underline{V}\,Pr\{V = \underline{V}/ S_l\} + VPr\,\{\bar{V}=V/S_l\} \quad \bar{}$$

We read the above as, the ask price at period l is the conditional expectation of V given that a trader wishes to buy from the market maker, and the bid price is the same, given that a trader wishes to sell
We use the Bayes Rule probability to determine

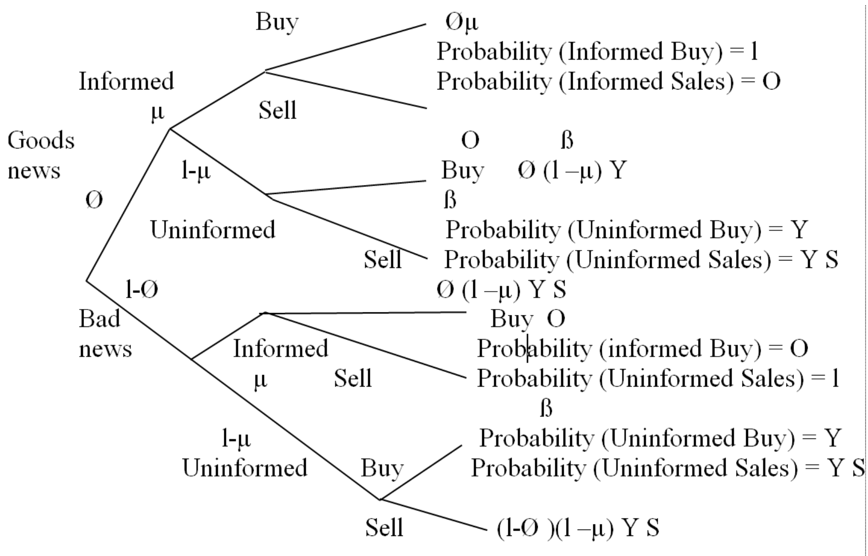
$$Pr\{V = \underline{V}/ S_l\} \text{ and } Pr\,\{V=V/\bar{S}_l\}$$

i.e. $$Pr\{V = \underline{V}/ S_l\} = \frac{Pr\{V = V\}\, Pr\{\, Sl/V = V\}}{Pr\{V = \underline{V}\}\, Pr\{S_l[/V = \underline{V}\,\} + Pr\{\bar{V} = V\}\, \bar{P}r\{S_l/V=V\}}$$

The rest probabilities can be done same way, i.e. $Pr\{V = \underline{V}/B_l\}$,

$$Pr\{V = \underline{\bar{V}}/v/S^l\} \text{ and } Pr\{V = \underline{\bar{V}}/B_l\}.$$

Alternatively, the above can be calculated using the probability structure of Trade, which is very simple; with a trader having a good news or bad news, be it informed or uninformed.



The Probability Structure of Trade:

Note:

Nature	**Traders**	**Traders**
Chooses Information	Learn information	decide to buys or sells

Ø is the probability the signal is good news
l – Ø is the probability the signal is bad news
µis the probability a trader is informed
l– µ is the probability a trader is uninformed
YB is the probability an uninformed trader buys
Ys is the probability an uninformed trader sells

Considering the large quantities of trading and price behaviour, the informed trader do trading in large quantities, as such the market maker's pricing policy in the separating equilibrium has this dual properties:

- If trades involve small quantity, there is no spread; as such market maker does not protect himself.
- If trades are in large quantity then, there is a spread; and the prices are:

$$b^* = V^* - \frac{\sigma v^2}{\bar{V} - \underline{V}} \left(\frac{\alpha u}{X^2 s(1-\alpha u) + S\alpha u} \right)$$

$$a^* = V^* + \frac{\sigma v^2}{\bar{V} - \underline{V}} \left(\frac{\alpha u}{X^2 s(1-\alpha u) + \alpha u(l-S} \right)$$

Where V^* is the expected value of V with $V \varepsilon [\underline{v}, \bar{v}]$
X is the fraction of uninformed traders who trade large quantity.
S is the probability that V is equal to $\underline{V}$
$\sigma^2 v$ is the variance of V
αu is the probability of informed trading; and depends on the probality of an information event and part of informed traders.

For b* and a* to constitute equilibrium, this condition must hold:

$$S^2/s' \underline{>} /+\alpha\mu S/X^2 s(l - \alpha\mu)$$

$$B^2/B' \underline{>} /+\alpha\mu(l\text{-}S)/X^2 B(l - \alpha\mu)$$

Where $S^2(B^2)$ represents the larger sell (buy) size

S'(B') represents the smaller sell (buy) size

Informed traders are guaranteed to trade a large quantity at "worse" price to have higher profit and not to trade a smaller quantity at a better price.

And with asymmetric information, price effect of a trade depends both on the trade and the current as well as sequence of past trades. In this case, prices are Martingales, i.e. there are not Markov. Because, if it is Markov, it only depends on current state. And such price process will be:

$$E(Pt+ l/Pt) = E(Pt + l/Pt \; ; Pt- l, \ldots\ldots\ldots Pl).$$

Informed and Uninformed Strategic Trader Models:

The informed traders have private information while the uninformed traders have no information advantage, they are liquidity motivated. When there is observed net order imbalance, the market makers doadjust his prices with both traders trading anonymously. In real life, information on sources and motivation of trades could have great effect on prices.

With the informed trader, we have the market maker's pricing strategy given as $p = P(x + \mu)$, thus, equilibrium price must satisfy

$$P(x + \mu) = E[V/X + \mu]$$

And with a given pricing rule, the informed trader order strategy must satisfy

$$E[\pi(X\,(*), P)/+\bar{V} = V] > E[\pi(X'\,(*), P)/\bar{V} = V], \text{ for each V.}$$

Note that the expected profit of strategy X (*) is greater than that of strategy X' (*) for the unformed trader.

In this model, there is equilibrium as noted by Kyle with the informed trader strategies and market maker given as:

$$X\,(v) = \beta(V - Po)$$

and

$$P(x + \mu) = Po + \lambda(x+ \mu)$$

Where $\beta = \left(\dfrac{6\mu^2}{\Sigma o}\right)^{1/2}$ and $\lambda = \frac{1}{2}\left(\dfrac{6\mu^2}{\Sigma o}\right)^{-1/2}$

The equilibrium in this model is derived with a straight forward application of the laws of conditional distribution of normally distributed random variables.

If $\varnothing = < \varnothing\,(l), \varnothing\,(2) >$ is distributed $N/\mu,\Sigma)$, and Σ is nonsingular, then

$$E\,(\varnothing^{(1)} / \varnothing^{(2)}) = \mu^{(1)} + \Sigma 12\, \Sigma 22^{-1}\, (\varnothing^{(2)} - \mu^{(2)})$$

We apply this by defining

$$\varnothing \;\Xi \begin{pmatrix} \underline{V} \\ Y \end{pmatrix} \quad \mu = \begin{pmatrix} E(\bar{v}) \\ E(x+\mu \end{pmatrix} = \begin{pmatrix} Po \\ \alpha+ \beta Po \end{pmatrix}$$

With $\bar{Y}$ as the aggregate (net) order flow and

$$\Sigma \;=\; \begin{pmatrix} \Sigma 12 \; \Sigma 12 \\ \Sigma 21 \; \Sigma 22 \end{pmatrix} = \begin{pmatrix} \Sigma o & \beta\Sigma o \\ \beta\Sigma o & 6\mu^2 + \beta^2\Sigma o \end{pmatrix}$$

(1) (2)

Using E (Ø / Ø), we state

$$E(V/Y) = \mu + E12^{(1)}\,\Sigma22^{-1}\,(Y-\mu)^{(2)}$$

$$= Po + \frac{\beta\Sigma o}{б\mu^2 + \beta^2\Sigma o}(Y - \alpha + \beta Po)$$

$$= Po + \lambda Y.$$

If the Market Maker believes that the informed trader will follow linear order strategy, x = β(V - Po). Then the Market Maker only sees x + µ and X, and he calls x + U to be Ø. Thus, Ø = x + µ = β(V - Po) + µ. When we rearrange the terms, we have,

$$Ø/\beta + Po = V + \mu/ß$$

If Z is equal to LHS $\therefore\; Z \;\Xi\; Ø/\beta + Po = V + \mu/ß$

With a known Z, the Marker Maker changes his beliefs about the asset's value r and gives a market-clearing price. The new price i.e. his new posterior mean is:

$$Pl = \frac{Po/\Sigma o + Z(\beta^2/б\mu^2)}{l/\Sigma o + \beta^2/б\mu^2}$$

and its variance is

$$Pl = (l/\Sigma o + \beta^2/б\mu^2)^{-1}$$

Substituting for ß and terms rearrange in the price equation, we have:

$$Pl = \frac{Poб\mu^2 + б\mu^2 Z}{б\mu^2 + б\mu^2} = \tfrac{1}{2}(Po+Z)$$

$$= \tfrac{1}{2}(Po + Ø/\beta + Po)$$

Note that Ø = X + µ,

Now, we have Pl = $Po + \frac{x+\mu}{2\beta} = Pl + \tfrac{1}{2}\left[\frac{б\mu^2}{\Sigma o}\right]^{-\frac{1}{2}}(x+\mu)$

$$= Po + \lambda(x+\mu)$$

Where $X = \tfrac{1}{2}\left[\frac{б\mu^2}{\Sigma o}\right]^{-\frac{1}{2}}$

This makes the Market Maker's price to be linear in the order flow.

Note that the informed trader's order strategy is:

$$X = \left[\frac{б\mu^2}{\Sigma o}\right]^{-\frac{1}{2}}(V - P)$$

and his ex ante profit is given by:

$$\pi = \tfrac{1}{2}(\Sigma o\, б\mu^2)^{-\frac{1}{2}}$$

The informed and uninformed traders are trading twice, thus the Market Makers price is:

$$P = \frac{Po + 1}{2\left(\frac{б\mu^2}{\Sigma o}\right)^{\frac{1}{2}}}\left(\left(\frac{б\mu^2}{\Sigma o}\right)\tfrac{1}{2}(v-Po) + \mu\right)$$

At period t, the informed order flow is:

$$Xt = \sum_{i=l}^{n} x_t^{i}$$

And the discretionary demand in period t is

$$\sum_{j=l}^{n} Y_t^{i}$$

Where Yj = Yj if jth discretionary trader trades in period t, and otherwise is O. Thus, the variance of total liquidity trading in period t is:

$$\Psi t \;\Xi\; var \quad \Sigma \left(\sum_{j=l}^{mj} Yt + zt \right)$$

The above depends on the strategic decisions of the discretionary trades with respect to when they will transact.

Thus, total order flow in period t, with ώt is given by:

$$ώt = \sum_{i=l}^{n\,i} xt + \sum_{i=l}^{m\,i} ŷt + Žt$$

The Market Maker is assume to use linear pricing rule

$$Pt = \bar{V} + \sum_{r=l}^{t} Sr + \lambda\, шt$$

Where шt is the total order flow in period t.

λ is the effect of order flow on the market price. Thus, market maker's order strategy shows both public and private information. And with optimal order strategy of informed trader i is given by:

$$\bar{X}^{i} = \beta\, (št + l + \bar{\Sigma} t\,)$$

t
i

Where $ßt = \sqrt{\dfrac{\Psi t}{nt[var(št+l) + Øl]}}$

An equilibrium value of λ is

$$\lambda t = \frac{var(št+l}{nt + l} \quad \sqrt{\frac{nt}{\Psi t[var(št+l) + Øt]}}$$

In conclusion, there is no model that is prefer to the other because of Admati and PFleiderer's assumption; that no reason for any period to be preferred to another; and that discretionary trade flows become informative over time; and this discretionary trader, trade earlier in the day.

Application of the O.R models to the Nigerian:
Capital Market:
Inventory Model and the Role of Price Formation:

In the Nigerian Capital Market (NCM), just as in other markets, the market makes who represent the volume traders uses the bid and asks prices to manage the in-balance in buy and sells orders, this make the market makers have effective control over the inventory. The inventory model plays a crucial role in price determination in the market. This model helps to track excessive inventory, that is, it reduces excess inventory; because some traders with required knowledge do selective trades at other dealers' prices

The Role of Information-Based Models 'Cum' Strategic Traders Model:

In the Nigerian Capital Market (NCM), we have many market participants, who have different and varied information about the market. This model considers the theory of market prices and its spread; and there is no room for inventory model. The model examines the market dynamics and gives insights to the price adjustment process Bagehot (1971). The Market Makers who are seen as the volume traders do not at periods have key information over the average trader such that the purchases and sales tends to follow fall and increase ion the bid prices respectively.

Calamia, (1999) is of the view that price adjusts to such factors as past prices and trades to reflect private information, inventory management and operating costs, dealers constrained with inventory costs and adverse information risks adjust quotes in response to observed transactions to reflect the information it conveys.

Market information is of great importance in the general markets. Information lies at the bottom of the debate over the relative efficiency of floor versus electronic trading systems. Madhavan (2000), put it this way, while floor based systems such as the Nigerian Stock Exchange and the New York Stock Exchange do not post customer limit order (except they represent the best quotes), on the other hand, electronic limit order book systems in place at the Nasdaq-NMS and Paris Bourse display current quotes as well as information on limit orders from the best quotes.

In the Nigerian Capital Market, the information available to the market participants is influenced by the allocation of the gains and losses to the trade; and price setting process is usually influenced by the quality and quantity of information in the hands of the market participants. The trade from an informed trader have a probability of πI and from uninformed trader, the probability is $1 - \pi I$.

In the Nigerian Capital Market, we have the informed traders who have private information and the uninformed trader with no information advantage. There is always an observed net order in-balance, and in such situation, the market makers to adjust their prices with both traders trading anonymously. In the Nigerian Capital Market, the information on sources and motivation of trades has great effect on prices. For the informed trader, the equilibrium price must satisfy $P(x + \mu) = E\,[V/X + \mu]$.

All models are okay for use in the Nigerian Capital Market, because Admati and Pfleiderer's assumes that there is no reason for any period to be preferred to another; and that discretionary trade flows become informative over time; and this discretionary trader, trade very early daily.

Summary, recommendations and conclusion summary:

In the Nigeria Capital Market, the formation of prices for securities is very important; the market participants are always faced with dual prices for the securities i.e. the Bid-ask prices.We are of the view that market participants are many and they carry out different and at time similar roles in the market. Really, common and advantageous to the uninformed traders are the two fold aspect of trading. And price continuity and stability is the goal of Market Maker and trading mechanism.

Recommendations:

This paper suggest that, in the Nigerian financial markers, use should be made of the models so as the boost the Market with respect to efficiency and reduction in bid-ask spreads which will bring to the citizens improvement in the social welfare. And in addition, for the models to work and operation effectively and efficiently, use should be made of modern technology with emphasis on real-time on-line electronic trade. If the education given to the broker and dealers is sound, it will lead to societal welfare improvement.

Conclusion:

We have attempted to show that an operations research model plays vital roles in the pricing financial assets and the up count of the market liquidity.

The broker and dealers in the Nigeria Capital Market should keep an eye in the operations research models – inventory models, information based models, and the strategic traders models of informed and uninformed used in this paper, so as to ensure effectiveness and efficiency in the Capital Market System.

This paper is of the view that the regulatory institutions should implement the recommendations. And this will make the Nigeria Financial Market to measure up with the global standard.

References:

[1] Admati, A andPFleiderer (1991).Sunshine Trading and Financial Market Equilibrium, Review of Financial Studies, 4(3), PP. 443-482.
[2] Amihud, Y and Mendelson, H. (1980). Dealership Market: Market Making with Inventory.
[3] Amihud, Y and Mendelson, H. (1987).Trading Mechanisms and stock Returns: AnEmpirical Investigations, Journal ofFinance 42, PP. 533-553.
[4] Bagehot, W. [Pseud] (1971). The Only Game in Town, Financial Analysts Journal 27, PP. 12 – 14, P. 22.
[5] Cohen, K. Maier, S. Schwartz, R. and Whitcomb, D. (1981).Transaction Costs, Order Placement Strategy, and Existence of Bid-Ask spread, Journal of Political Economy 80, PP. 281-305.
[6] Copeland, T. and Galai, D. (1983).Information Effects and the Bid-Ask Spread, Journal of Finance 38, PP. 1457-1469.
[7] Demsetz, H. (1968). The Cost of Transacting, Quarterly Journalof Economics 82, PP. 33-53.
[8] Easley, D. and O'Hara, M. (1987a).Price, Trade Size, and Information in SecuritiesMarkets, Journal of Financial Economics 19, PP. 69-90.
[9] Easley, D. and O'Hara, M. (1987b).Large Trade Volume and Market Efficiency, Working Paper, Cornell University.
[10] Easley, D. and O'Hara, M. (1992a).Time and the Process of Security Price Adjustment, Journal of Finance 47, PP. 577-606.
[11] Easley, D. and O'Hara, M. (1992b).Adverse Selection and Large Trade Volume: The Implications for Market Efficiency, Journal of Financial and Quantitative Analysis 27,PP. 185-208.
[12] Garman, M. (1976). Market Microstructure, Journal of Financial Economics 3, PP. 257-275.
[13] Glosten, L. and Mitgrom, P. (1985).Bid, Ask and Transaction Prices in a SpecialistMarket with Heterogeneously Informed Traders, Journal of Financial Economics 13,PP. 71-100.
[14] Ho, T. and Stoll, H. (1981) Optimal Dealer Pricing Under Transactions and Return Uncertainty, Journal of Financial Economics 9, PP. 47-73.
[15] Ho, T. and Stoll, H. (1983). The Dynamics of Dealer Markets under Competition, Journal of Finance 38, PP. 1053-1074.
[16] Kyle, A. S. (1984). Market Structure, Information, Future Markets and Price Formation, in International Agricultural Trade: Advanced Readings in price Formation, Market Structure and Price Instability, ed. By Story, G. Schmitz, A. and Sarris, A. (Westview Press, Boulder and London).
[17] Madhavan, A. (1992).Trading Mechanisms in Securities Markets, Journal of Finance 47, PP. 607-652.
[18] O'Callaghan (1993). The Structure of the World Gold Market, Occasional Paper No. 105, International Monetary Fund, Washington, D.C.
[19] O'Hara, M. and Oldfield, G. (1986).The Microeconomics of Market Making, Journal of Financial and Quantitative Analysis 21, December, PP. 361- 376.
[20] O'Hara, M. (1995). Market Microstructure Theory, Cambridge, MA: Blackwell Publishers.
[21] Osamwonyi, I. O. and Igbinosa, S. O. (2007).Market Microstructure Theory: Synthesis and Application, Journal of Finance and Banking Review
[22] (FBR) Vol.1, No.1, August, PP. 54-73.leiderer, P. (1984). The Volume of Trade and the Variability of Prices: A Framework for Analysis in Noisy Rational Expectations Equilibria, working Paper, Stanford University.
[23] Wikipedia Free: Market Microstructure (http//www:en.wikipedia.Encyclopedia accessed October 15, 2009). Wiki/market_micro-structure.

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