MULTICRITERIA METHODOLOGIES IN
DECISION MAKING PROCESS IN FINANCE

Doris MADHI,
PhD.Candidate,
University “Aleksandër Xhuvani”
Faculty of Economics, Rruga Kozma Naska
Elbasan, Albania

ABSTRACT

Nowadays the decision making process in different areas considering multiple influent factors has become a priority. The financial institutions have been facing multiple risks during the recent years, usually interacting risks which makes necessary an efficient decision making, in order to optimize the capital allocation and increase profitability. Different authors have given a great contribution at the purpose, considering different variables which mainly lead to the same conclusion. Thus, decision makers always run the risk of being influenced by their own experience and preferences and a unique and optimal solution can’t be reached if multiple criteria are not considered. The existing theories of Multi Criteria Decision Aid suggest the consideration of multiple criteria quantitative and qualitative in order to select the most influential ones and to create a ranking from the best to the worst. The merit of these methods (ELECTRE, PROMETHEE and MURAME) consists in the complex structuring allowance and the consideration of the interacting variables. These methods follow the exploiting phase, which allows the creation of an outranking performance matrix of variables based not only on the manager’s preferences, but also on objective evaluation criteria. The main merit associated to these methods is related to their versatility and broader use in different areas of decision making.

Keywords: Decision making, Multi Criteria Decision Aid, Ranking, ELECTRE, PROMETHEE.
INTRODUCTION:

It is obvious that every institution has to take prudential decisions regarding their financial statements’ components without precluding the possibility to increase profitability. Usually the decision making process is related to ‘fund term allocation’ and ‘optimization of the capital structure’. Every decision taken in the financial environment aims to realize optimization, in order to choose the best alternative. It is merely difficult to optimize the decisions in the financial field, as often, the objectives to achieve, contrast each other. This is the reason why the decision making methodologies aiming to optimization are quite limited, as objectives are different depending on long or short term priorities. According to (Zapounidis, 1999), there are three principal reasons leading to a prospective change in order to integrate optimization methods:

1. The author presumes that the optimum point of view of the decision maker makes him often deviate from the real decision core problem.
2. The human decision making process is hardly influenced by the decision maker’s experience, knowledge and preferences.
3. It is illusionary to speak about optimization in financial decision making, as multiple criteria has to be considered.

According to these considerations, the financial decision makers have to dispose tools which permit the simultaneous consideration of different and sometimes conflicting criteria to advance their decision making process. Multi criteria analysis, often called Multiple Criteria Decision Making (MCDM) by the American School and Multi Criteria Decision Aid (MCDA) by the European School (Zapounidis, 2009), is a set of methods which allow the aggregation of several evaluation criteria in order to choose, rank, sort or describe a set of alternatives. It also deals with the study of the activity of decision aid to a well identified decision maker (i.e. individual, firm, organization, etc.) (Zopounidis, 1999). The multi criteria approach leads to four important results (Roy, 1985):

i. Identification of the best alternative or alternative sets, considering the other alternatives;
ii. Constructions of a ranking ordering of the alternatives from the best to the worst;
iii. Classification in homogeneous groups for the alternatives;
iv. Identification of each alternative distinguishing features and performs their description based on these features.

(Zopounidis, 1999) concludes that the main advantages that the MCDA methods provide to decision making are summarized as follows:

- The complex structuring is an important issue for the decision maker and these methods make it possible;
- The introduction of both qualitative and quantitative criteria, considering the influence some qualitative factors have on quantitative variables (ratios). These interactions are quite difficult to be measured by optimization methods, but widely integrated in MCDA methods, which do not make any distinction between qualitative or quantitative criteria.
- Evaluation transparency, allowing good argumentation for financial decisions: the MCDA models allow the decision maker to participate actively in the decision making process and focus on the modeling of his preferences. Such interactive methods allow to the decision maker to become familiar with the used method and explore the core of the considered problem. These features also give him the due argument to support the taken decision.
- Introduction of sophisticated and realistic techniques: these methods are based not only on sound scientific fundaments but also on realistic assumptions, easy to be implemented in practice, to effectuate the financial decision making. For sure other methods do exist, but they are not based often on realistic assumptions, which constitutes their weak implementation point.

The MCDA methods are very versatile and easy to be practically used in numerous different fields such as marketing, medicine, human resources, environmental and energy management and production systems.

FUNDAMENTS OF MULTI CRITERIA DECISION AID: LITERATURE REVIEW:

In (Zopounidis & Doumpos, 2002) mentioned that there are two issues specified during every classification or sorting methodology such as follows (1) Development of a criteria aggregation model form which can be used for the purpose of classification/sorting and (2) the methodology which is employed in order to define the model parameters. The above issues apply to almost all the MCDA classification/sorting methodologies.

The MCDA models should initially consider the determination of a set of \( n \) alternatives \( A= \{a_1,...,a_i,...,a_m\} \) and a set of \( n \) criteria \( C=\{c_1,...,c_j,...,c_n\} \). Every considered alternative should be evaluated based on these
determinations. Every day, we face problems to solve and our decision regarding the best alternative has to be based on the different characteristics which every problem holds. For instance, when we decide to buy a car, the decision should be based on different characteristics concerning items such as price, fuel consumption, life span, color, maximum speed etc. We have to make an evaluation and a decision considering not only quantitative characteristics (ex. Price) but also qualitative characteristics (ex. color). The decision should be taken on basis of the tradeoff between different alternatives and pairs of criteria created.

Regarding the methodologies used to implement this decision making process, several authors distinguish different kind of methods. According to (Roy, 1985), the proposal is based on three main categories: 1) unique criterion-based approach disregarding incomparability, usually represented by a utility function; 2) the outranking approach accepting incomparability; 3) the interactive local judgment approach, considering trial error interactions.

According to (Scharlig, 1996), the multi criteria methods can be distinguished as complete, partial and local aggregation methods. The first category is based on the so called Multiple Attribute Utility Theory (MAUT). The family of MAUT methods consists of aggregating different criteria into a function, which has to be maximized. This theory allows complete compensation between criteria, i.e. the gain on one criterion can compensate the lost on another (Keeney & Raiffa, 1976). There are three building blocks for their procedures. First is the performance matrix and the second is procedures to determine whether criteria are independent of each other or not. The third consists of ways of estimating the parameters in a mathematical function which allow the estimation of a single number index, \( U \), to express the decision maker’s overall valuation of an option in terms of the value of its performance on each of the separate criteria.

There is an uncertain built by this model into the account directly into the decision support models which enables the attributes communicate with one another in a very simple and additive fashion. It does not assume mutual independence of preferences. In certain circumstances, it can be important to build into the analysis one or both of these factors, but often in practice it may be better to ignore them in order to allow a simpler and more transparent decision support to be implemented more quickly, by a wider range of users and for a larger set of problem types.

Nevertheless, these models have been criticized merely based on the maximization of a utility function, which is not easy to be estimated and calculated in practice. Based on these considerations, the outranking methods were developed by (Roy, 1968). These methods provide the construction of an outranking relation between variables, thus the assessment of the outranking degree of an alternative \( a_i \) that outranks another alternative \( a_j \) based also on the decision maker’s preferences. This aggregation phase is followed by the exploitation phase, which uses the build outranking relation to create a preferences matrix through several procedures.

The outranking models are represented by ELECTRE and PROMETHEE. According to Roy (1974), the outranking relation is generally a pure mathematical relation. He assumes that the outranking relation is represented by a binary relation that enables the assessment of the outranking degree of an alternative \( a_i \) over an alternative \( a_j \). The outranking relation allows concluding that \( a_i \) outranks \( a_j \) if there are enough arguments to confirm that \( a_i \) is at least as good as \( a_j \) (concordance), while there is no essential reason to refute this statement (discordance). Thus, these methods are used to eliminate the dominated alternatives, but also using some weights (reflecting event the decision makers preferences) to give more influence to some criteria towards others.

It is also important to consider the way how this outranking model performs when the given alternatives are not comparable between it (not to be confused with indifference) because, for example, at the assessing period some important data regarding an alternative is missing. In this case, the outranking methods are just formal and the judgment of the alternatives performance is not so accurate, just because the information is missing. As this situation may often happen in practice, this constitutes a weak point of the outranking models, which can confuse the incomparability (because of missing data), with indifference relation between alternatives. The main concern voiced about the outranking approach is that, it depends on arbitrary definitions of what precisely constitutes outranking and how the threshold parameters (concordance or discordance) are set and later manipulated by the decision maker.

The third decision making model refers to more recent studies and consists on the combination between calculation and decision makers preferences, which aims to introduce even some subjective issues related to the decision maker.

\(^1\) Dominance occurs when one alternative performs at least as well as another on all criteria and strictly better than the other on at least one criterion. In principle, one alternative might dominate all others, but in practice this is unlikely. When it does occur, it is helpful to ask if there is some advantage of the dominated alternative that is not represented by the criteria; this may reveal new criteria that have been overlooked. Dominance is more likely just to enable the decision-making team to eliminate dominated alternatives from further consideration.
It is important to consider, however, that all Multi Criteria Decision Making Procedures are characterized by the interaction between calculations and decision maker’s preferences; it is the decision maker which defines the problem through the chosen alternatives based on their criteria. The decision maker plays a very important role also on the determination of the final solution of the problem, as the mathematical solution is always aggregated by further information to choose the best fit alternative.

In later stages, the base of the current paper discusses much on Multi Criteria Ranking Method (MURAME) used by Corazza, Funari and Siviero (2008). The final objective of this method is building a final outranking of all alternatives from the best to worst. The considered alternatives may also be ranked in homogeneous group, thus the method can be used even like a sorting or classification method. The nascence of this model is due to the combination of the most known methodologies ELECTRE III and PROMETHEE II frameworks. Thus, the initial phase of MURAME follows the ELECTRE III methodology, trying to calculate the dominance of a certain alternative $a_i$ towards another alternative $a_j$; the second phase, the exploitation phase, refers to PROMETHEE II method, which aggregates the different dominance grades to build a final ranking score.

METHODOLOGY:

This paper creation is based on the theoretical research of methodologies which lead to the decision making process in finance and further. It is important during different empirical studies to have a proficient theoretical basis. Multiple theories and authors are considered in order to have an accurate view of the whole process. In the paper, a logical line of suggested decision making methods follow each other from the simplest one to the more complex.

FINDINGS AND DISCUSSIONS:

THE ELECTRE APPROACH:

This method elaboration is due to Bernard Roy during the late 1960’s. The main objective in this approach speaks about the necessity to give a confidence grade, thus to provide enough arguments that the alternative $a_i$ is at least as good as the alternative $a_j$. This assumption is valid if sufficient elements affirm it.

The ELECTRE (ELimination Et Choix Traduisant la REALité) method considers a set of $m$ alternatives (objective) $A=\{a_1,...,a_i,...,a_m\}$ described by a set of $n$ criteria (subjective) $G=\{c_1,...,c_j,...,c_n\}$. The considered alternatives are the subjects to be evaluated and may be represented by firms, institutions, investment projects, and other possible alternative subjected to any type of study. Further, we have to define $g_{i,j}$ that indicates the performance of each criteria $c_j$ related to each $a_i$ alternative. Thus, a performance matrix can be build.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$g_{1,1}$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>$g_{i,1}$</td>
</tr>
<tr>
<td>$a_m$</td>
<td>$g_{m,1}$</td>
</tr>
</tbody>
</table>

Table 1: The Performance Matrix

We have to consider that:

- $g_{i,j}$ is a real number; $\forall g_{i} \in A \exists g_{k} \in A \text{ such that } g_{i} \geq g_{k}$ is at least as $g_k$, considering the criteria $c_j$.

This point of view, considering the classical models, leads to three possible alternatives for each pair of alternatives:

- $g_{i,j} > g_{k,j}$, which means that each $i$ alternative is better than any $k$ alternative;
- $g_{i,j} = g_{k,j}$, which means equality for each pair of alternatives, thus the decision maker is indifferent in his choice between two alternatives;
- $g_{k,j} > g_{i,j}$, which means that each $k$ alternative is better than each $i$ alternative.

However, in real conditions, pure certainty does not exist regarding the absolute tradeoff an alternative towards another. For instance, let’s consider two cups of coffee, one having little bit more sugar than the other; even one is sweeter than the other, is difficult to note the difference. Thus, even in the decision process, sometimes the difference may be irrelevant or difficult to perceive by the decision maker.
If the decision is situated on both first cases, it indicates a weak preference and if it is situated between the first and third case, it indicates incomparability between them. This is a relevant problem to be solved not only in ELECTRE methods, but also in other multi criteria methods. This problem is solved through introduction of two different thresholds called indifference threshold and preference threshold. These thresholds allow considering an uncertainty decision zone, before passing from a strong preference situation to an indifference situation. Based on these models, it may be assumed that a tradeoff exists between alternatives if their difference is greater than a certain quantity \( p \); this quantity represents the preference threshold, which allows the decision maker to determine exactly the preferred alternatives (as in case of strong preference condition). On the other side, if the difference between two alternatives is less than \( p \), two situations may arise: the weak preference and the indifference condition.

The last case, may occur even when a pair of alternatives aren’t exactly equal, thus the alternatives are indifferent when their difference is less than a \( q \) quantity, necessarily less than \( p \), called ‘indifference threshold’. The decision maker’s passage from the indifference condition to a strong preference condition is called weak preference. The weak preference may occur when the difference between two alternatives is greater than the indifference threshold and less than the preference threshold.

The introduction of a double threshold preference can lead to one of these cases:
- \( ai P ak \), represents the case in which each \( i \) alternative is better than each \( k \) alternative and \( P \) denotes the preference relation;
- \( ai Q ak \), represents the case in which a weak preference relation is established between \( ai \) and \( ak \);
- \( ai I ak \), represents an indifference relation between alternatives or a not relevant difference between \( gi, j \) and \( gk,j \).

The quantities such as \( pj \) and \( qj \) denote the preference and indifference thresholds according to any \( j \) criteria respectively. The \( p \) and \( q \) values should consider the decision maker’s preferences, thus should be determined in a subjective way. The \( pj \) threshold should be greater than the \( qj \) threshold, \( pj > qj \) and these values may be constant or differ based on the considered criteria. In case of different values, it assumes a linear form \( \alpha+\beta x \), where \( x \) denotes the evaluation based on a criteria and \( \alpha+\beta \) are constant values. It doesn’t exist a general accepted method to calculate the thresholds values, so different considerations have been done.

According to (Maystre, Pictet, & Simos, 1994) the indifference threshold represents the maximum error threshold related to a criterion. According to (Roy, 1968) these thresholds are considered symmetric and denoted with “\( e \)”. Once established a maximum value for the criteria denoted with \( N \), the threshold value range can be \( N+e \) and \( N-e \), where \( e=\alpha+\beta N \). This interpretation of the threshold value marks a not constant value of the threshold, but rather a criterion parameterized value. The threshold \( qj \) value is expressed as follows:

\[
q[j(ai)] = \alpha + \beta gj(ai)
\]

**Equation 1:**

We denote with \( gj(ai) \) the evaluation of the \( ai \) alternative based on \( gj \) criterion. The parameter \( \beta \) ranges between 1 and -1 and \( \alpha \) may be any real number, such that \( \alpha+\beta gj(ai) \) has a positive value. The preference threshold is expressed as:

\[
p[jg(ai)] = \frac{2[\alpha + \beta gj(ai)]}{1 - \beta}
\]

**Equation 2:**

The ELECTRE methods introduce another threshold value, called the ‘veto threshold’, denoted with \( vj \), which aims to determine the discordance indicators. This value is always greater than the preference threshold considering any criterion, thus \( vj > pj \).

Considering one of the mentioned criteria, the \( k \) criteria for instance, the multi criteria methods leads to evaluate the alternatives as follows:
- \( ai \) is indifferent to \( ak \) if \( gij=gk \), \( \forall j\neq k \), even in case \( gi.k \neq gk.k \);
- \( ai \) is strongly preferred to \( ak \) if \( gij=gk \), \( \forall j\neq k \), but their difference is relevant.

This last expression affirms the result which can be obtained from evaluating a pair of alternatives, with different performances only, if one criterion is considered and the other ones remain equal. These alternatives are indifferent only, if the difference between \( ai \) and \( ak \) is not relevant. Once this difference is relevant, the alternative \( ai \) is strongly preferred to the alternative \( ak \), considered the \( j \) criterion, even when other criteria remains equal. Thus, it is important to note the importance of the established thresholds and each criterion in case of overall evaluation.
The mentioned considerations are valid for all decision making methods based on ELECTRE. As mentioned before, our interest topic in this study is related to MURAME, which first evaluation step refers to a variant of ELECTRE, called ELECTRE III, based on the outranking relation construction through alternatives. The outranking phase consists on the construction of a confidence matrix which contains for each pair of alternatives \((ai,ak)\) an outranking measure, a value that represents the credibility in the affirmation that “\(ai\) is at least as good as \(ak\)” (Corazza, Funari, & Siviero, 2014). This evaluation should be effectuated for each pair of alternatives and leads to the construction of two indexes called the ‘concordance index’ and the ‘discordance index’. The concordance index is calculated for each pair of alternatives and represents the dominance of an alternative to another, based on a certain criterion. On the other side, the discordance index expresses the opposite relation, thus it allows to measure how much the hypothesis \(ai\) dominates \(ak\) is not satisfied. Thus, the concordance index aims to the credibility of this affirmation for most part of the considered criteria, and the discordance index aims the opposite. Given a pair of alternatives \((a_i, a_k)\), it is necessary to calculate a local concordance index, which represents the credibility of the affirmation, considering a criteria \(c_j\). Thus, the local concordance index should be calculated for each pair of alternatives related to every criterion; given \(n\) alternatives and \(m\) criteria, the number of local concordance indexes will be \((n-1)\times n\times m\). This index is expressed as follows:

\[
C_j(ai, ak) = \begin{cases} 
1 & \text{if } gkj \leq gij + qj \\
0 & \text{if } gkj \geq gij + pj \\
gij - gkj + pj & \text{otherwise} \\
pj - qj & \text{otherwise}
\end{cases}
\]

**Equation 3:**

The concordance index ranges from 0 to 1, which are also its minimum and maximum value that express a strong preference condition. In fact, when the index \(C_j(ai, ak)\) assumes a value equal to one, the alternative \(ai\) is not dominated by the alternative \(ak\) and when it assumes the value zero means that \(ak\) is preferred to \(ai\). On the other side, the intermediate values between 0 and 1 assumed by the index represent a weak preference between alternatives, thus the decision maker is uncertain about the better alternative to decide. Further, it is necessary to determine the local discordance index; to verify how much the expression \(ai\) dominates \(ak\) according to the criteria \(cj\) is not satisfied. This index should be calculated for each pair of alternatives \((ai, ak)\) according to each considered criteria. This index is expressed as follows:

\[
D_j(ai, ak) = \begin{cases} 
0 & \text{if } gkj \leq gij + pj \\
1 & \text{if } gkj \geq gij + vj \\
gkj - gij - pj & \text{otherwise} \\
vj - pj & \text{otherwise}
\end{cases}
\]

**Equation 4:**

This index reaches its minimum value when the difference between \(ak\) and \(ai\) is less than the preference threshold. In fact, in this case \(ak\) is not strongly preferred to \(ai\) according to \(cj\) criteria. On the other side, a new value \(vj\) appears in this formula. This value is called veto threshold and aim to consider the case in which an alternative is worse than another, considering a given criteria. To realize this objective, it is necessary that the veto threshold assumes a higher value than the preference threshold, thus \(vj > pj\).

The discordance index assumes the value equal at one when the difference between \(ai\) and \(ak\) alternatives is greater than the veto threshold. In this case, the veto threshold is used to reject the fact that the alternative \(ai\) is at least as good as \(ak\). Concluding, the \(j\) criteria assumes the veto on the fact that \(ai\) is better than \(ak\). The discordance index assumes also values which range between zero and one, thus gives a discordance grade to the fact that \(ai\) is at least as good as \(ak\). This condition is verified when the difference between the alternatives is greater than the preference threshold but less than the veto threshold, thus not so relevant to give veto power to the considered criteria.

The local concordance index, as mentioned before evaluates the performance of a given alternative towards another, according to one criterion. Thus, it becomes necessary to aggregate the local concordance and discordance indexes in order to consider all the criteria simultaneously. This aggregation leads to the construction of an outranking index \(O(ai, ak)\) for each pair of alternatives. To construct the outranking index, we should first determine a global concordance threshold for each pair of alternatives, which consists in a weighted average of the respective local concordance index.
Equation 5:
As notable form the formula each local concordance index is weighted by \( w_j \), which represents the importance given to \( j \) criteria. Given the global concordance index we can calculate the outranking index as follows:
\[
O(ai, ak) = \begin{cases} 
C(ai, ak) & \text{if } \sum_{j=1}^{n} w_j C_j(ai, ak) \leq C(ai, ak) \forall j \\
C(ai, ak) \prod_{j \in K} \frac{1 - D_j(ai, ak)}{1 - C(ai, ak)} & \text{otherwise}
\end{cases}
\]

Equation 6:
The outranking formula shows that if the local discordance index is less than the local accordance index, for each criterion, it is representative of the outranking index. On the other side, this value has to be multiplied by a factor smaller than one, every time each the local discordance index is greater than the concordance index, considering each criterion.

The criteria for which \( D_j(ai, ak) > C(ai, ak) \) compose the subset \( K \). Notice that, if there exists even only one criterion for which there is maximum discordance (i.e. \( D_j(ai, ak) = 1 \)), then the outranking index is equal to 0. This entails that, if for one of the given criteria the alternative \( ai \) is “worse” than the alternative \( ak \), then it is not more possible consider \( ai \) at least as “good” as \( ak \) although this was true for all the remaining criteria. Under this point of view, the above described outranking approach is considered prudential.

The outranking index construction is followed by exploitation phase, called distillation phase, thus to create a total preorder of the alternatives. This phase is not considered in the MURAME METHOD. The successive phase of MURAME derives from the PROMETHEE II method, in order to create a total order after the outranking phase and classify the alternatives in a descending order.

THE PROMETHEE METHOD:

The PROMETHEE methods were developed by (Brans P. & Vincke P., 1985) as outranking methods. In this work, we are interested in PROMETHEE II method, which serves to complete the MURAME method second outranking phase, aiming to create a total order of the preferences (differently, PROMETHEE I, gives only a partial raking of the alternatives) (Brans P. & Vincke P., 1985). These methods were proposed by the authors to avoid the difficulties of a practical exploitation of the ELECTRE approach regarding the fixation of concordance and discordance thresholds and the credibility degree to be linked to the decision maker’s choice of preferences. Obviously, this approach allows decision to consider multiple characteristics of the alternatives. The weak and unrealistic point regarding other outranking methods is linked with the lack of quantification of difference between the performances of the alternatives pairs, so to have a net distinction of strong preference.

On the other side, even the indifference condition is transitive. In the literature, some studies have tried to settle these issues, introducing some extensions to the criteria aiming to create a larger area of hesitation between preference and indifference (pseudo criterions) and to consider a larger area of indifference (quasi criterions). These extensions to the criteria have been also applied by the PROMETHEE authors, resulting simpler and practical compared to the necessity of fixing concordance and discordance thresholds in ELECTRE.

(Brans P. & Vincke P., 1985) have also provided to introduce some further extension to the criteria in PROMETHEE approach. The extension is based on the introduction of a preference function giving the preference of the decision-maker for an action \( a \) with regard to \( b \). This function will be defined separately for each criterion; its value will be between 0 and 1. The indifference of the decision maker will be greater for each criterion.

Let us consider a multi criteria problem as defined in which each criterion having to be maximized. Let \( f(.) \) be a particular criterion and \( a \) and \( b \) two particular actions of \( K \). The associated preference function \( P(a, b) \) of \( a \) with regard to \( b \) will be defined as:
\[
P(a, b) = \begin{cases} 
0 & \text{if } f(a) < f(b) \\
p[f(a), f(b)] & \text{if } f(a) > f(b)
\end{cases}
\]

Equation 7:
The preference function will depend on the difference between the values \( f(a) \) and \( f(b) \). The authors have developed six different types of extensions which may cover the practical applications. For each criterion, only a few parameters (maximum two) have to be identified by the decision-maker. This seems an easy task in view of the fact that each parameter has a real economic meaning. Compared to ELECTRE approach, in
PROMETHEE approach the preference and indifference thresholds are not mandatory, but they depend on the preference model of the decision maker. Also, it is not included the determination of the veto threshold able to outrank an alternative when its performance is notably lower compared to another.

For each pair of alternatives $ai, ak$ we can denote:

$$\pi (ai, ak) = \sum_{j=1}^{k} Pj(ai, ak)wj$$

$$\pi(ak, ai) = \sum_{j=1}^{k} Pj(ai, ak)wj$$

**Equation 8:**
Where, $\pi (ai, ak)$ represents the dominance grade of the alternative $ai$ to the alternative $ak$, the quantity $Pj$ is the preference index and $wj$ the weight assigned to each criterion based on their importance. These quantities values range from 0 to 1. In fact, if $\pi (ai, ak)$ is equal to 0, means that the quantity $\pi(ak, ai)$ is equal to , thus the alternative $ak$ performance is better than $ai$ performance according to the considered criteria.

After the determination of the preference index we can proceed to determine the outranking flows, which will determine the domination power of an alternative towards another. Each alternative may be characterized by an outgoing flow and inflow as follows:

$$\varphi^+(ai) = \sum_{k \neq i} \pi(ai, ak)$$

**Equation 9:**

$$\varphi^-(ai) = \sum_{k \neq i} \pi(ak, ai)$$

**Equation 10:**

The outranking flow above represents respectively the strength and weakness of the alternative $ai$. In fact, the outranking positive flow indicates the grade of dominance of the alternative $ai$: greater the value $\varphi^+(ai)$, greater the $ai$ dominance towards other alternatives and smaller $\varphi^-(ai)$ indicates that $ai$ is less dominated.

At this point we can proceed to the total alternatives ranking step. As in PROMETHEE I it is not provided a total ranking of the alternatives, but only a partial one we will refer to PROMETHEE II, which is the basis also for the outranking construction in MURAME approach. This step consists on the determination of a net outranking flow, as the difference between outranking inflows and outflows. It is expressed as follows:

$$\varphi(ai) = \varphi^+(ai) - \varphi^-(ai) \forall i$$

**Equation 11:**

Since now we have illustrated the methods which are used to apply further the MURAME Approach.

**The MURAME Approach**

In the MURAME approach the outranking indexes are calculated as the ELECTRE III indexes, considering the concordance and discordance information. The outranking index aims to determine how much the alternative $ai$ outranks the alternative $ak$, considering all the criteria as follows:

$$O(ai, ak) = \begin{cases} 
C(ai, ak) & \text{if } Dj(ai, ak) \leq C(ai, ak) \forall j \\
C(ai, ak) \prod_{j \in K} \frac{1 - Dj(ai, ak)}{1 - C(ai, ak)} & \text{otherwise}
\end{cases}$$

**Equation 12:**

Note that:

$$C(ai, ak) = \frac{\sum_{j=1}^{n} wjCj(ai, ak)}{\sum_{j=1}^{n} wj}$$

$C(ai, ak)$ represents the weighted mean of the local concordance indexes in which $wj$ indicates the weight associated to criterion $cj$ and $K$ denotes the subset of deponents of the criteria for which $Dj (ai, ak) > C (ai, ak)$.

The second step to fulfill follows the PROMETHEE II lines and consists on the construction of a total ranking of the alternatives. In this phase an outflow and an inflow are calculated in order to evaluate the strength and the weakness of the alternative $ai$ over the other alternatives as follows:

$$\varphi^+(ai) = \sum_{k \neq i} O(ai, ak)$$
Equation 13:
\[
\varphi^-(ai) = \sum_{k \neq i} O(ai, ak)
\]

Equation 14:
In order to obtain a total preorder of the alternatives the final net flow for each \( ai \) alternative is calculated as the difference between the outflows and the inflows as follows:
\[
\varphi(ai) = \varphi^+(ai) - \varphi^-(ai)
\]

Equation 15:
Given the outflows and the inflows two alternatives preorders may be built: a descending order and an ascending preorder. Given a pair of alternatives \((ai, ak)\), they can be compared creating a partial preorder as follows:
\[
\begin{align*}
aiPak & \text{ if } \varphi^+(ai) \geq \varphi^+ ak \text{ or } \varphi^-(ai) \leq \varphi^- ak, \text{ where } P \text{ indicates a strong preference}; \\
& \text{at least one relation should be strong} \\
aiP^- ak & \text{ if } \varphi^+(ai) \leq \varphi^+ ak \text{ or } \varphi^-(ai) \geq \varphi^- ak, \text{ where } P^- \text{ indicates a weak preference}; \\
& \text{at least one relation should be strong} \\
aiIak & \text{ if } \varphi^+(ai) = \varphi^+ ak \text{ or } \varphi^-(ai) = \varphi^- ak \text{ where } I \text{ indicates indifference between } ai \text{ and } ak
\end{align*}
\]

All other different cases indicate incompatibility between the given alternatives. The final total outranking process could be obtained through the utilization of the total net flows. In conclusion, the MURAME approach characteristics are summarized as follows:
1. The decision maker preference order is built based on the strong preference condition, indifference and weak preference;
2. The outranking relation is built through the calculation of discordance and concordance indexes;
3. The inflows and outflows calculation allows building not only a partial preorder, but also a total order of the alternatives.

CONCLUSION:

In practice, it is quite difficult to implement any decision making process which is not influenced by the decision’s maker experience, knowledge or preferences. Since every taken decision has to consider multiple criteria, it becomes necessary to have decision making tools which allow integrating preferences and objectivity of evaluation. Usually, the decision making process has to be based on a utility maximizing function, difficult to be implemented in practice. The theoretical steps suggested by different authors include the identification of the best alternative or alternative sets, considering the other alternatives; the construction of a ranking ordering of alternatives, the classification in homogeneous groups and identification of each alternative based on their performance.

The MCDA methods are built for this purpose and their advantages are related to the complex structuring of the alternatives, the consideration of qualitative and quantitative criteria and their interactions, their evaluation transparency and an objective integration to the decision maker preferences. The MCDA methods are very versatile and easy to be practically used in numerous different fields such as marketing, medicine, human resources, environmental and energy management and production systems.

In this resume, we have seen ELECTRE III Method, PROMETHEE II Method and MURAME, which is a combination of both. The first method was built to give a confidence grade to the decision, thus to provide enough arguments that the alternative \( a_i \) is at least as good as the alternative \( a_j \). The construction of a preferences matrix of alternative sets is completed trough the introduction of preference and indifference (discordance and concordance) thresholds in order to consider the interaction between alternatives. The last phase of this method is the construction of a ranking between alternatives from the best to the worst. The weak point consists in the thresholds fixation, which could be not realistic or often objective. Thus the PROMETHEE Method was built to avoid the difficult implementation of ELECTRE and the unrealistic decisions, through the creation of extensions to the distinction of the alternatives and their performances in strong, indifferent and weak. It consists in the outranking construction as the difference between inflows (dominance) and outflows (weaknesses) between the considered alternatives.

The construction of the MURAME Method, which is a combination of previous methods, allows avoiding the encountered weaknesses. It consists in making a decision based on the strong, indifferent and weak preference.
conditions. The process is completed following the discordance and concordance indexes calculation and creating a total preorder and ranking of the alternatives.

REFERENCES:


----