ROBUSTNESS OF NESTED BALANCED INCOMPLETE BLOCK DESIGNS AGAINST UNAVAILABILITY OF TWO BLOCKS

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ABSTRACT

Mating designs are the study of progenies developed through various methods like Diallel Cross plans which are subjected to Incomplete Block Designs. The concept of robustness in designs has been studied and available in the literature. The effects of missing blocks on Complete Diallel Cross designs are examined in this study. A-efficiencies based on non -zero eigenvalues suggest that these designs are fairly robust. The investigation shows that Nested Balanced Incomplete Block Designs are fairly robust in terms of efficiency. In this paper, the robustness of Nested Balanced Incomplete Block Design when two blocks are lost has been discussed.

Keywords: Nested Balanced Incomplete Block Design; Efficiency of residual design; Mating Design; Youden Square Design; Latin Square Design.

INTRODUCTION:

Diallel Mating Design has become most popular among the breeders and geneticists. It consists of set of all possible single crosses among a given set of *n* lines. Griffing (1956) made further contributions to Diallel Mating Design by developing suitable models and methods of analysis. Schmidt (1919) introduced the concept of Diallel crossing as a means of comparing the breeding values of parents. It was further adopted in other situation by Comstock and Robinson (1952). Curnow (1963) discussed the construction of Diallel crossed experiment using RBD yields a large number of crosses so later on various other authors develop the construction of Diallel Crossed experiment using Balanced Incomplete Block Design, PBIBD and so on. After getting the cross, one has to test and verify the best or promising variety of crop. Diallel crosses plan, are further classified into two types of plans namely,

1) Complete Diallel Cross plan.(CDC) and
2) Partial Diallel Cross plan. (PDC)

Diallel Cross experiment is said to be a Complete Diallel Cross (CDC) design if all potential crosses occur at least once in the design, even though they need not be replicated the same number of times, see Ghosh and Desai (1999). When the number of lines is increased, the number of crosses becomes so large that there may not be enough experimental material to accommodate a Complete Diallel Cross design. In these situations, a Partial Diallel Cross (PDC) design involving fewer crosses may be chosen to estimate the general combining ability of the *p* lines, as considered by Gilbert (1958) and Kempthorne and Curnow (1961).

Ghosh and Desai (1998, 1999) obtained the robustness of Complete Diallel Crosses Plan against the unavailability of one block and also for those plans, which have unequal number of crosses in a block. Further, Ghosh and Biswas (2000) also pointed out the robustness for Complete Diallel Crosses Plan, which are binary, balanced against the loss of one block. Das and Kageyama (1992) showed that Balanced Incomplete Block Designs and extended Balanced Incomplete Block Designs are fairly robust against the unavailability of $s(s \leq k)$ observations in any block. While any Youden Design and Latin Square Designs are found to be fairly robust against the loss of any one column.

Prescott and Mansson (2004) investigated the effect of missing observations on complete diallel cross designs. They examined the robustness of CDC using BIBD and PBIBD. A-efficiencies, based on average variances of the elementary contrasts of the line effects, suggest that Complete Diallel Cross Design is fairly robust against the unavailability of observations. Mansson and Prescott (2004) examined the robustness of CDC against the loss of a block of observations using BIBD and PBIBD. They found a simple generalized inverse for the information matrix of the line effects, which allows evaluation of expressions for the variances of the line-effects differences with and without the missing block. A-efficiencies, based on average variances of the elementary contrasts of the line-effects, suggest that CDC is fairly robust against the unavailability of a block.

Preece (1967) has introduced a case of two way elimination of heterogeneity, one nested within the other. He also has introduced a Nested Balanced Incomplete Block Design and gave method of construction of Nested Balanced Incomplete Block Design. Further, he listed a table of available Nested Balanced Incomplete Block Designs. Some experimental units are the half leaves of plants. There may be more treatments than there are suitable half leaves per plant, where as there may be variation between plants, between leaves within plants and between half leaves within leaves. Here, both leaves and plants can be thought as system of blocks, one system Nested within the others.

This paper looks into the Robustness of CDC plan when two blocks are lost from a design using Nested Balanced Incomplete Block Design. C^* matrix and its non-zero eigenvalues are computed with its corresponding multiplicity and its efficiency. It shows that CDC plans are fairly robust against the loss of two blocks from a design. Nested BIB Design with unavailability of two blocks is considered for different parametric values. Corresponding C^* matrices and their non-zero eigenvalues with multiplicities are computed for each set of parameters and it appears that the CDC designs are fairly robust against the unavailability of two blocks.

The robustness criteria against the unavailability of data are: *(i)* to get the connectedness of the residual design; *(ii)* to have the variance balance of the residual design; *(iii)* to consider the A-efficiency of residual design for the robustness study. So far, robustness of Incomplete Block Designs and complete block designs are carried out against loss of $s(s \leq k)$ observations in one block.

In this investigation, consider a connected CDC plan D. Let D^* be the residual design obtained when one or more observations are lost. Assume D^* to be connected. In this case, the criterion of robustness against the unavailability of one or more observations is the overall A-efficiency, of the residual design D^*, given by,

$$e(s) = \frac{\text{Sum of reciprocals of non - zero eigenvalues of C}}{\text{Sum of reciprocals of non - zero eigenvalues of C}^*}$$

$$e(s) \;=\; \frac{\phi_2(s)}{\phi_1(s)} \tag{1.1}$$

C – MATRIX OF COMPLETE DIALLEL CROSSES PLAN:

We know that for any block design C matrix can be defined as,

$$C = rI_v - \frac{NN'}{k}$$

Now C matrix is given as,

$$C = \begin{bmatrix} r(k-1) & \lambda & \lambda \\ \lambda & r(k-1) & \lambda \\ \lambda & \lambda & r(k-1) \end{bmatrix} - \frac{\begin{bmatrix} r(k-1)^2 & \lambda(k-1) & \lambda(k-1) \\ \lambda(k-1) & r(k-1)^2 & \lambda(k-1) \\ \lambda(k-1) & \lambda(k-1) & r(k-1)^2 \end{bmatrix}}{\frac{k(k-1)}{2}}$$

$$C = \left[\frac{\lambda v(k-2)}{k}\right] \left[I_v - \frac{E_{vv}}{v}\right]$$

$$C = \Theta \left[I_v - \frac{E_{vv}}{v}\right]$$

So, $\Theta = \left[\frac{\lambda v(k-2)}{k}\right]$

The non - zero eigenvalues of C_D^* matrix and its corresponding multiplicity of Complete Diallel Cross Design can be given by, $\Theta = \left[\frac{\lambda v(k-2)}{k}\right]$ with multiplicity (v-1), respectively.

ROBUSTNESS OF NESTED BALANCED INCOMPLETE BLOCK DESIGN AGAINST THE UNAVAILABILITY OF TWO BLOCKS:

Complete Diallel Crosses Plan was considered a Nested Balanced Incomplete Block Design with parameters $v = p$, b_1, b_2, r, k_1, k_2, λ_1, λ_2, m. Suppose two blocks of a Nested Balanced Incomplete Block Design with two blocks are lost. Under this situation, the following four cases will occur:

Case *i*: Unavailability of two blocks where the number of common lines between two blocks are zero.

Case *ii:* Unavailability of two blocks where the number of common lines between two blocks are one.

Case *iii*: Unavailability of two blocks where the number of common lines between two blocks are two.

Case *iv*: Two same blocks which are repeated is lost

For all four cases when two blocks are lost from Nested Balanced Incomplete Block Design, efficiency factor is depending upon the common number of lines between two lost blocks. The efficiency for all four cases when common number of lines between two lost blocks are 0, 1, 2, 3, . . . , (k-1), k respectively are studied. Here, the robustness criterion of Nested Balanced Incomplete Block Design was further discussed for the different value of common number of lines between two blocks.

Case (*i*): Unavailability of two blocks where the number of common lines between two blocks are zero.

Consider a Nested Balanced Incomplete Block Design D with parameters $v = p$, b_1, b_2, r, k_1, k_2, λ_1, λ_2, m. Let two blocks be lost. Call this design as a residual design assumes a residual design D^* is a connected design. Let the blocks be b_i and b_j, let their zero line is common between two lost blocks i.e. $\eta(b_i \cap b_j) = 0$. Each line which is present in the two lost blocks will be replicated (r-1) times. All remaining line will be replicated r times in design.

Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{jj} = \frac{(r-1)(k-1)}{k}$, where j denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{ll} = \frac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in following ways, which is said as λ_1, λ_2. Pattern of λ_i $(i = 1, 2)$ are as follows,

1. $\lambda_1 = (\lambda - 1)$, for those lines, which are present in two lost blocks.
2. $\lambda_2 = \lambda$, for remaining pair of lines.

The C^* matrix of design D^* can be written as,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v - k)I_k - (\lambda - 1)J_{kk} & -\lambda J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{kk} & (\lambda v - k)I_k - (\lambda - 1)J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{(v-2k)k} & -\lambda J_{(v-2k)k} & \lambda v(I_{v-2k} - v^{-1}J_{(v-2k)(v-2k)}) \end{bmatrix}$$

The non – zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(\lambda v - k)}{k}$, with multiplicity $2(k\text{-}1)$.
2. $\frac{\lambda v}{k}$, with multiplicity $(v\text{-}2k\text{+}1)$.

THEOREM 1:

Nested Balanced Incomplete Block Designs with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m$ are fairly robust against the unavailability of two blocks, where number of common line between two lost blocks are zero, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda v - k)(v - 1)}{(\lambda v - k)(v - 2k + 1) + 2\lambda v(k - 1)} \tag{1.2}$$

PROOF:

Without loss of generality, let two blocks be lost from design D where number of common line between two blocks is zero i.e. $\eta(b_i \cap b_j) = 0$, C^* matrix of the residual design is given by,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v - k)I_k - (\lambda - 1)J_{kk} & -\lambda J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{kk} & (\lambda v - k)I_k - (\lambda - 1)J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{(v-2k)k} & -\lambda J_{(v-2k)k} & \lambda v(I_{v-2k} - v^{-1}J_{(v-2k)(v-2k)}) \end{bmatrix}$$

The non – zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(\lambda v - k)}{k}$, with multiplicity $2(k\text{-}1)$.
2. $\frac{\lambda v}{k}$, with multiplicity $(v\text{-}2k\text{+}1)$.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{1.3}$$

Where $\phi_2(s)$ = sum of reciprocals of non-zero eigenvalues of C matrix of design D

and $\phi_1(s)$ = sum of reciprocals of non-zero eigenvalues of C^* matrix of design D^*.

$$\text{That is, } \phi_2(s) = \frac{k(v-1)}{\lambda v(k-2)} \quad \text{and} \tag{1.4}$$

$$\phi_1(s) = \frac{k(\text{v - 2k} + 1)}{\lambda v} + \frac{2(k-1)k}{(\lambda v - k)} \tag{1.5}$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda v - k)(v-1)}{(\lambda v - k)(v - 2k + 1) + 2\lambda v(k-1)} \tag{1.6}$$

EXAMPLE 1:

Let D represent the Nested Balanced Incomplete Block Design with parameters
$v = 12, b_1 = 22, b_2 = 44, r = 11, k_1 = 6, k_2 = 3, \lambda_1 = 5$. Design D is given by,

Table 1: 12 lines of NBIB design of Bhatt(2008)

Block	NBIB design						Crosses in the NBIB design		
1	1	2	3	5	8	12	1 × 12	2 × 8	3 × 5
2	2	3	4	6	9	12	2 × 12	3 × 9	4 × 6
3	3	4	5	7	10	12	3 × 12	4 × 10	5 × 7
4	4	5	6	8	11	12	4 × 12	5 × 11	6 × 8
5	1	5	6	7	9	12	1 × 12	5 × 9	6 × 7
6	2	6	7	8	10	12	2 × 12	6 × 10	7 × 8
7	3	7	8	9	11	12	3 × 12	7 × 11	8 × 9
8	1	4	8	9	10	12	1 × 12	4 × 10	8 × 9
9	2	5	9	10	11	12	2 × 12	5 × 11	9 × 10
10	1	3	6	10	11	12	1 × 12	3 × 11	6 × 10
11	1	2	4	7	11	12	1 × 12	2 × 11	4 × 7
12	4	6	7	9	10	11	4 × 11	6 × 10	7 × 9
13	1	5	7	8	10	11	1 × 11	5 × 10	7 × 8
14	1	2	6	8	9	11	1 × 11	2 × 9	6 × 8
15	1	2	3	7	9	10	1 × 10	2 × 9	3 × 7
16	2	3	4	8	10	11	2 × 11	3 × 10	4 × 8
17	1	3	4	5	9	11	1 × 11	3 × 9	4 × 5
18	1	2	4	5	6	10	1 × 10	2 × 6	4 × 5
19	2	3	5	6	7	11	2 × 11	3 × 7	5 × 6
20	1	3	4	6	7	8	1 × 8	3 × 7	4 × 6
21	2	4	5	7	8	9	2 × 9	4 × 8	5 × 7
22	3	5	6	8	9	10	3 × 10	5 × 9	6 × 8

Two blocks containing block 1 and block 12 are lost and number of common line between two blocks is zero. C^* matrix of the residual design is given by,

$$6C^* = \begin{bmatrix} 50 & -4 & -4 & -4 & -4 & -4 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & 50 & -4 & -4 & -4 & -4 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & 50 & -4 & -4 & -4 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & -4 & 50 & -4 & -4 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & -4 & -4 & 50 & -4 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & -4 & -4 & -4 & 50 & -5 & -5 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & 50 & -4 & -4 & -4 & -4 & -4 \\ -5 & -5 & -5 & -5 & -5 & -5 & -4 & 50 & -4 & -4 & -4 & -4 \\ -5 & -5 & -5 & -5 & -5 & -5 & -4 & -4 & 50 & -4 & -4 & -4 \\ -5 & -5 & -5 & -5 & -5 & -5 & -4 & -4 & -4 & 50 & -4 & -4 \\ -5 & -5 & -5 & -5 & -5 & -5 & -4 & -4 & -4 & -4 & 50 & -4 \\ -5 & -5 & -5 & -5 & -5 & -5 & -4 & -4 & -4 & -4 & -4 & 50 \end{bmatrix}$$

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{54}{6}$, with multiplicities 10.
2. $\frac{60}{6}$, with multiplicities 1.

The overall A- efficiency of the design is,

$$e(s) = 0.908257$$

CASE (*II*): UNAVAILABILITY OF TWO BLOCKS WHERE NUMBER OF COMMON LINES BETWEEN TWO BLOCKS ARE ONE:

Consider a Nested Balanced Incomplete Block design with parameters $v = p$, b_1, b_2, r, k_1, k_2, λ_1, λ_2, m. Let two blocks be lost. Call this design as a residual design and assume that the residual design D^* is a connected design. Let the blocks be b_i and b_j, let one line be common between two lost blocks i.e. $\eta(b_i \cap b_j) = 1$. Here this line is repeated (r-2) times. Similarly those lines which are present in the two lost blocks but are not common will be replicated (r-1) times. The remaining lines will be replicated r times in design. Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{ii} = \dfrac{(r-2)(k-1)}{k}$, where i denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{jj} = \dfrac{(r-1)(k-1)}{k}$, where j denotes those lines which are presents in both the lost blocks but are distinct.
3. $C_{ll} = \dfrac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in following three ways, which is said as λ_1, λ_2, λ_3. Pattern of λ_i (i = 1, 2, 3) are as follows,

1. $\lambda_1 = (\lambda - 1)$, for those lines, which are present in two lost blocks.
2. $\lambda_2 = (\lambda - 1)$, for those lines, which are present in two lost blocks.
3. $\lambda_3 = \lambda$, for remaining lines.

The C^* matrix of design D^* can be written as,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v-2k+1)I_1-(\lambda-1)J_{11} & -(\lambda-1)J_{1(k-1)} & -(\lambda-1)J_{1(k-1)} & -\lambda J_{1(v-2k+1)} \\ -(\lambda-1)J_{(k-1)1} & (\lambda v-k)I_{(k-1)}-(\lambda-1)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-2k+1)} \\ -(\lambda-1)J_{(k-1)1} & -\lambda J_{(k-1)(k-1)} & (\lambda v-k)I_{(k-1)}-(\lambda-1)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-2k+1)} \\ -\lambda J_{(v-2k+1)1} & -\lambda J_{(v-2k+1)(k-1)} & -\lambda J_{(v-2k+1)(k-1)} & \lambda v(I_{(v-2k+1)}-v^{-1}J_{(v-2k+1)(v-2k+1)}) \end{bmatrix}$$

The non–zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\dfrac{(\lambda v-2k+1)}{k}$, with multiplicity 1.
2. $\dfrac{(\lambda v-k)}{k}$, with multiplicity 2(k-1).
3. $\dfrac{\lambda v}{k}$, with multiplicity (v-2k+1).
4. $\dfrac{(\lambda v-1)}{k}$, with multiplicity 1.

THEOREM 2:

Nested Balanced Incomplete Block Designs with parameters $v = p$, b_1, b_2, r, k_1, k_2, λ_1, λ_2, m are fairly robust against the unavailability of two blocks, where number of common line between two blocks is one, provided the overall efficiency of the residual design is given by

$$e(s) = \frac{(v-1)(\lambda v-2k)(\lambda v-1)(\lambda v-2k+1)}{(\lambda v-2k)(\lambda v-1)((v-2k+1)(\lambda v-2k+1)+\lambda v)+(\lambda v-2k+1)\lambda v(2(k-2)(\lambda v-1)+(\lambda v-2k))} \qquad (1.7)$$

PROOF:

Without loss of generality, let two blocks be lost from design D where number of common line between two blocks is one i.e. $\eta(b_i \cap b_j) = 1$, C^* matrix of the residual design is given by,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v-2k+1)I_1-(\lambda-1)J_{11} & -(\lambda-1)J_{1(k-1)} & -(\lambda-1)J_{1(k-1)} & -\lambda J_{1(v-2k+1)} \\ -(\lambda-1)J_{(k-1)1} & (\lambda v-k)I_{(k-1)}-(\lambda-1)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-2k+1)} \\ -(\lambda-1)J_{(k-1)1} & -\lambda J_{(k-1)(k-1)} & (\lambda v-k)I_{(k-1)}-(\lambda-1)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-2k+1)} \\ -\lambda J_{(v-2k+1)1} & -\lambda J_{(v-2k+1)(k-1)} & -\lambda J_{(v-2k+1)(k-1)} & \lambda v(I_{(v-2k+1)}-v^{-1}J_{(v-2k+1)(v-2k+1)}) \end{bmatrix}$$

The non – zero eigenvalues of C* matrix with their corresponding multiplicities are,

1. $\dfrac{(\lambda v-2k+1)}{k}$, with multiplicity 1.

2. $\dfrac{(\lambda v - k)}{k}$, with multiplicity $2(k\text{-}1)$.

3. $\dfrac{\lambda v}{k}$, with multiplicity $(v\text{-}2k\text{+}1)$.

4. $\dfrac{(\lambda v - 1)}{k}$, with multiplicity 1.

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{1.8}$$

Where $\phi_2(s)$ = sum of reciprocals of non-zero eigenvalues of C matrix of design D

and $\phi_1(s)$ = sum of reciprocals of non-zero eigenvalues of C^* matrix of design D^*.

That is, $\phi_2(s) = \dfrac{k(v-1)}{\lambda v(k-2)}$ and (1.9)

$$\phi_1(s) = \frac{k(\mathrm{v} - 2\mathrm{k} + 1)}{\lambda v} + \frac{k}{(\lambda v - 2\mathrm{k} + 1)} + \frac{2(k-2)k}{(\lambda v - k)} + \frac{k}{(\lambda v - 1)} \tag{1.10}$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(v-1)(\lambda v - 2k)(\lambda v - 1)(\lambda v - 2k + 1)}{(\lambda v - 2k)(\lambda v - 1)((v - 2k + 1)(\lambda v - 2k + 1) + \lambda v) + (\lambda v - 2k + 1)\lambda v(2(k-2)(\lambda v - 1) + (\lambda v - 2k))} \tag{1.11}$$

EXAMPLE 2:

Let D represent the Nested Balanced Incomplete Block Design with parameters $v = 9$, $b_1 = 12$, $b_2 = 24$, $r = 8$, $k_1 = 6$, $k_2 = 3$, $\lambda_1 = 5$. Design D is given by,

Table 2: 9 lines of NBIB design of Bhatt(2008)

Block	NBIB design						Crosses in the NBIB design		
1	1	3	5	7	8	9	1 × 9	3 × 8	5 × 7
2	1	3	5	7	8	9	1 × 9	3 × 8	5 × 7
3	1	3	5	7	8	1	1 × 1	3 × 8	5 × 7
4	1	3	5	7	8	2	1 × 2	3 × 8	5 × 7
5	1	3	5	7	8	6	1 × 6	3 × 8	5 × 7
6	2	4	6	9	1	3	2 × 3	1 × 4	6 × 9
7	2	4	6	7	4	6	2 × 6	4 × 4	6 × 7
8	2	4	6	4	1	9	2 × 9	1 × 4	4 × 6
9	2	4	6	6	8	7	2 × 7	4 × 8	6 × 6
10	2	4	9	3	2	5	2 × 5	2 × 4	3 × 9
11	7	8	9	8	2	3	3 × 7	2 × 8	9 × 8
12	9	6	5	4	5	9	9 × 9	5 × 6	4 × 5

Two blocks containing block 1 and block 7 are lost and number of common lines between two blocks is one. C^* matrix of the residual design is given by,

$$3C^* = \begin{bmatrix} 36 & -4 & -4 & -4 & -4 & -5 & -5 & -5 & -5 \\ -4 & 38 & -4 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & 38 & -5 & -5 & -5 & -5 & -5 & -5 \\ -4 & -5 & -5 & 38 & -4 & -5 & -5 & -5 & -5 \\ -4 & -5 & -5 & -4 & 38 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & 40 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & 40 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & -5 & 40 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & -5 & -5 & 40 \end{bmatrix}$$

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{40}{3}$, with multiplicities 1.
2. $\frac{42}{3}$, with multiplicities 2.
3. $\frac{44}{3}$, with multiplicities 1.
4. $\frac{45}{3}$, with multiplicities 4.

The overall A- efficiency of the design is,

$$e(s) = 0.979976$$

CASE (III): UNAVAILABILITY OF TWO BLOCKS WHERE NUMBER OF COMMON LINES BETWEEN TWO BLOCKS ARE TWO:

Consider a Nested Balanced Incomplete Block Design D with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m.$ Let two blocks be lost. Call this design as a residual design and assume that the residual design D^* is a connected design. Let the blocks be b_i and b_j, let number of common lines between two blocks be two, i.e. $\eta(b_i \cap b_j) = 2$. Similarly those lines which are present in the two lost blocks but are not common will be replicated (r-1) times. The remaining lines will be replicated r times in design.

Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{ii} = \frac{(r-2)(k-1)}{k}$, where i denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{jj} = \frac{(r-1)(k-1)}{k}$, where j denotes those lines which are presents in both the lost blocks but are distinct.
3. $C_{ll} = \frac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in the following three ways, which is said as λ_1, λ_2, λ_3. Pattern of λ_i (i = 1, 2, 3) are as follows,

1. $\lambda_1 = (\lambda\text{-}2)$, for those lines, which are present in two lost blocks.
2. $\lambda_2 = (\lambda\text{-}1)$, for those lines, which are present in two lost blocks.
3. $\lambda_3 = \lambda$, for remaining lines.

The C^* matrix of design D^* can be written as,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v - k + 1)I_2 - (\lambda)J_{22} & -(\lambda - 1)J_{2(k-1)} & -\lambda J_{2\,(v-k-1)} \\ -(\lambda - 1)J_{(k-1)2} & (\lambda v - 2k)I_{(k-1)} - (\lambda - 2)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-k-1)} \\ -\lambda J_{(v-k-1)\,2} & -\lambda J_{(k-1)(v-k-1)} & \lambda v(I_{v-k-1} - v^{-1}J_{(v-k-1)\,(v-k-1)}) \end{bmatrix}$$

The non – zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(\lambda v - k + 1)}{k}$, with multiplicity 1.
2. $\frac{(\lambda v - 2k)}{k}$, with multiplicity $(k\text{-}2)$.
3. $\frac{(\lambda v - k - 1)}{k}$, with multiplicity 1.
4. $\frac{\lambda v}{k}$, with multiplicity $(v\text{-}k\text{-}1)$.

THEOREM 3:

Nested Balanced Incomplete Block Design with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m$ are fairly robust against the unavailability of two blocks, where number of common lines between two blocks are two, i.e. $\eta(b_i \cap b_j) = 2$, provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(v-1)(\lambda v - 2k)(\lambda v - k + 1)(\lambda v - k - 1)}{(\lambda v - k + 1)(\lambda v - k - 1)((v - k - 1)(\lambda v - 2k) + \lambda v(k-2)) + 2\lambda v(\lambda v - k)(\lambda v - 2k)} \qquad (1.12)$$

PROOF:

Without loss of generality, let two blocks be lost from design D where number of common line between two blocks is two i.e. $\eta(b_i \cap b_j) = 2$, C^* matrix of the residual design is given by,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v - k + 1)I_2 - (\lambda)J_{22} & -(\lambda-1)J_{2(k-1)} & -\lambda J_{(k-1)(v-k-1)} \\ -(\lambda-1)J_{(k-1)2} & (\lambda v - 2k)I_{(k-1)} - (\lambda-2)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-k-1)} \\ -\lambda J_{(v-k-1)(k-1)} & -\lambda J_{(k-1)(v-k-1)} & \lambda v(I_{(v-k-1)} - v^{-1}J_{(v-k-1)(v-k-1)}) \end{bmatrix}$$

The non – zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\dfrac{(\lambda v - k + 1)}{k}$, with multiplicity 1.
2. $\dfrac{(\lambda v - 2k)}{k}$, with multiplicity ($k$-2).
3. $\dfrac{(\lambda v - k - 1)}{k}$, with multiplicity 1.
4. $\dfrac{\lambda v}{k}$, with multiplicity (v-k-1).

Further, overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{1.13}$$

Where $\phi_2(s)$ = sum of reciprocals of non-zero eigenvalues of C matrix of design D

and $\phi_1(s)$ = sum of reciprocals of non-zero eigenvalues of C^* matrix of design D^*.

$$\text{That is, } \phi_2(s) = \frac{k(v-1)}{\lambda v(k-2)} \quad \text{and} \tag{1.14}$$

$$\phi_1(s) = \frac{k(\text{v - k - 1})}{\lambda v} + \frac{k}{(\lambda v \text{ - k + 1})} + \frac{(k-2)k}{(\lambda v - 2k)} + \frac{k}{(\lambda v \text{ - k - 1})} \tag{1.15}$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(v-1)(\lambda v - 2k)(\lambda v - k + 1)(\lambda v - k - 1)}{(\lambda v - k + 1)(\lambda v - k - 1)((v - k - 1)(\lambda v - 2k) + \lambda v(k-2)) + 2\lambda v(\lambda v - k)(\lambda v - 2k)} \tag{1.16}$$

EXAMPLE 3:

Let D represent the Nested Balanced Incomplete Block Design with parameters
$v = 9$, $b_1 = 12$, $b_2 = 24$, $r = 8$, $k_1 = 6$, $k_2 = 3$, $\lambda_1 = 5$. Design D is given by,

Table 3: 9 Lines of Nbib Design Of Bhatt(2008)

Block	Nbib Design						Crosses In The Nbib Design		
1	1	3	5	7	8	9	1 × 9	3 × 8	5 × 7
2	1	3	5	7	8	9	1 × 9	3 × 8	5 × 7
3	1	3	5	7	8	1	1 × 1	3 × 8	5 × 7
4	1	3	5	7	8	2	1 × 2	3 × 8	5 × 7
5	1	3	5	7	8	6	1 × 6	3 × 8	5 × 7
6	2	4	6	9	1	3	2 × 3	1 × 4	6 × 9
7	2	4	6	7	4	6	2 × 6	4 × 4	6 × 7
8	2	4	6	4	1	9	2 × 9	1 × 4	4 × 6
9	2	4	6	6	8	7	2 × 7	4 × 8	6 × 6
10	2	4	9	3	2	5	2 × 5	2 × 4	3 × 9
11	7	8	9	8	2	3	3 × 7	2 × 8	9 × 8
12	9	6	5	4	5	9	9 × 9	5 × 6	4 × 5

Two blocks containing block 1 and block 9 are lost and number of common lines between two blocks is two. C^* matrix of the residual design is given by,

$$3C^* = \begin{bmatrix} 38 & -5 & -4 & -4 & -4 & -5 & -5 & -5 & -5 \\ -5 & 38 & -4 & -4 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & 36 & -3 & -5 & -5 & -5 & -5 & -5 \\ -4 & -4 & -3 & 36 & -5 & -5 & -5 & -5 & -5 \\ -4 & -5 & -5 & -5 & 40 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & 40 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & 40 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & -5 & 40 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & -5 & -5 & 40 \end{bmatrix}$$

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{39}{3}$, with multiplicities 1.
2. $\frac{41}{3}$, with multiplicities 1.
3. $\frac{43}{3}$, with multiplicities 2.
4. $\frac{45}{3}$, with multiplicities 5.

The overall A- efficiency of the design is,

$$e(s) = 0.964097$$

CASE (IV): TWO SAME BLOCKS WHICH ARE REPEATED IS LOST:

Consider a Nested Balanced Incomplete Block Design D with parameters $v = p, b_1, b_2, r,$ $k_1, k_2, \lambda_1, \lambda_2, m$. Let two same blocks which are repeated be lost from a design. Call this design as a residual design and assume that the residual design D^* is connected design. Let two lost blocks be b_i and b_j, and the number of common line between two lost blocks are k. Each line which is present in the two lost block will be replicated $(r$-2$)$ times while remaining line will be replicated r times.

Let C^* be the information matrix of design D^*. For this design D^*, the diagonal element of C^* matrix are as follows,

1. $C_{ii} = \frac{(r-2)(k-1)}{k}$, where i denotes those lines which are present in both the lost blocks but are distinct.
2. $C_{ll} = \frac{r(k-1)}{k}$, where l denotes the remaining lines.

Similarly, in the residual design, pair of lines occurs together in the following two ways, which is said as λ_1, λ_2. Pattern of λ_i $(i = 1, 2)$ are as follows,

1. $\lambda_1 = (\lambda\text{-}2)$, for those lines, which are common in two lost blocks.
2. $\lambda_2 = \lambda$, for remaining line.

The C^* matrix of design D^* can be written as,

$$k(k\text{-}2)C^* = \begin{bmatrix} (\lambda v - 2k)I_k - (\lambda - 2)J_{kk} & -\lambda J_{(k)(v-k)} \\ -\lambda J_{(v-k)(k)} & \lambda v(I_{(v-k)} - v^{-1}J_{(v-k)(v-k)} \end{bmatrix}$$

The non – zero eigenvalues of C^* matrix with their corresponding multiplicities are,

1. $\frac{(\lambda v - 2k)}{k}$, with multiplicity $(k$-1).
2. $\frac{\lambda v}{k}$, with multiplicity $(v$-$k)$

THEOREM 4:

Nested Balanced Incomplete Block Designs with repeated blocks and with parameters $v = p, b_1, b_2, r, k_1, k_2, \lambda_1, \lambda_2, m$ are fairly robust against the unavailability of two blocks (two same repeated blocks) such that number of common lines between two blocks are k provided the overall efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda v - 2k)(v-1)}{\lambda v(k-1) + (v-k)(\lambda v - k)} \tag{1.17}$$

PROOF:

Without loss of generality, let two blocks be lost C^* matrix of the residual design is given by,

$$kC^* = \begin{bmatrix} (\lambda v - 2k)I_k - (\lambda - 2)J_{kk} & -\lambda J_{(k)(v-k)} \\ -\lambda J_{(v-k)(k)} & \lambda v(I_{(v-k)} - v^{-1}J_{(v-k)(v-k)} \end{bmatrix}$$

The non – zero eigenvalues of C* matrix with their corresponding multiplicities are,

1. $\frac{(\lambda v - 2k)}{k}$, with multiplicity ($k$-1).
2. $\frac{\lambda\ v}{k}$, with multiplicity (v-k)

Finally overall A-efficiency is calculated as,

$$e(s) = \frac{\phi_2(s)}{\phi_1(s)} \tag{1.18}$$

Where $\phi_2(s)$ = sum of reciprocals of non-zero eigenvalues of C matrix of design D

and $\phi_1(s)$ = sum of reciprocals of non-zero eigenvalues of C* matrix of design D^*.

That is, $\phi_2(s) = \frac{k(v-1)}{\lambda v(k-2)}$ and (1.19)

$$\phi_1(s) = \frac{k(\text{v - k})}{\lambda v} + \frac{(k-1)k}{(\lambda v - 2k)} \tag{1.20}$$

Finally, A- efficiency is given by,

$$e(s) = \frac{(\lambda v - 2k)(v-1)}{\lambda v(k-1) + (v-k)(\lambda v - k)} \tag{1.21}$$

EXAMPLE 4:

Let D represent the Nested Balanced Incomplete Block Design with parameters $v = 9$, $b_1 = 12$, $b_2 = 24$, $r = 8$, $k_1 = 6$, $k_2 = 3$, $\lambda_1 = 5$. Design D is given by,

Table 4: 9 lines of NBIB design of Bhatt(2008)

Block	NBIB Design						Crosses In The NBIB Design		
1	1	3	5	7	8	9	1 × 9	3 × 8	5 × 7
2	1	3	5	7	8	9	1 × 9	3 × 8	5 × 7
3	1	3	5	7	8	1	1 × 1	3 × 8	5 × 7
4	1	3	5	7	8	2	1 × 2	3 × 8	5 × 7
5	1	3	5	7	8	6	1 × 6	3 × 8	5 × 7
6	2	4	6	9	1	3	2 × 3	1 × 4	6 × 9
7	2	4	6	7	4	6	2 × 6	4 × 4	6 × 7
8	2	4	6	4	1	9	2 × 9	1 × 4	4 × 6
9	2	4	6	6	8	7	2 × 7	4 × 8	6 × 6
10	2	4	9	3	2	5	2 × 5	2 × 4	3 × 9
11	7	8	9	8	2	3	3 × 7	2 × 8	9 × 8
12	9	6	5	4	5	9	9 × 9	5 × 6	4 × 5

Two Blocks containing blocks 1 and 2 are lost twice from the design, C^* matrix of the residual design is given by,

$$3C^* = \begin{bmatrix} 36 & -3 & -3 & -5 & -5 & -5 & -5 & -5 & -5 \\ -3 & 36 & -3 & -5 & -5 & -5 & -5 & -5 & -5 \\ -3 & -3 & 36 & -5 & -5 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & 40 & -5 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & 40 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & 40 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & 40 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & -5 & 40 & -5 \\ -5 & -5 & -5 & -5 & -5 & -5 & -5 & -5 & 40 \end{bmatrix}$$

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{39}{3}$, with multiplicities 2.
2. $\frac{45}{3}$, with multiplicities 6.

The overall A- efficiency of the design is,

$$e(s) = 0.962963$$

Table 5: Efficiency table when two blocks is lost from a Nested Balanced Incomplete Block Design

D. No	v	b_1	b_2	r	k_1	k_2	λ	Case *i* e(s)	Case *ii* $e(s)$	Case *iii* $e(s)$	Case *iv* $e(s)$
1.	5	5	10	4	4	2	3	0.928571	0.92562	0.92562	0.916667
2.	7	7	14	6	6	2	5	0.980198	0.979886	0.979886	0.978947
3.	7	7	14	6	6	3	5	0.941176	0.907733	0.938008	0.935484
4.	8	14	28	7	4	2	3	0.974684	0.974071	0.974071	0.972222
5.	9	18	36	8	4	2	3	0.980392	0.979976	0.979976	0.978723
6.	9	12	24	8	6	3	5	0.965517	0.946139	0.964097	0.962963
7.	9	9	18	8	8	2	7	0.99187	0.991802	0.991802	0.991597
8.	9	9	18	8	8	4	7	0.951613	0.917121	0.949316	0.948276
9.	10	15	30	9	6	2	5	0.990826	0.990727	0.990727	0.990431
10.	10	15	30	9	6	3	5	0.972414	0.957014	0.971399	0.970588
11.	10	10	30	9	9	3	8	0.982979	0.973869	0.982602	0.982301
12.	6	15	30	10	4	2	6	0.977011	0.976662	0.976662	0.97561
13.	11	11	22	10	10	2	9	0.995893	0.995872	0.995872	0.995807
14.	11	11	22	10	10	2	9	0.995893	0.995872	0.995872	0.995807
15.	12	33	66	11	4	2	3	0.989418	0.989255	0.989255	0.988764
16.	12	22	44	11	6	2	5	0.993769	0.993714	0.993714	0.993548
17.	12	22	44	11	6	3	5	0.981221	0.970867	0.980652	0.980198
18.	7	21	42	12	4	2	6	0.983607	0.983395	0.983395	0.982759
19.	13	39	78	12	4	2	3	0.991071	0.990945	0.990945	0.990566
20.	13	26	52	12	6	2	3	0.991071	0.990945	0.990945	0.990566
21.	13	26	52	12	6	3	3	0.972973	0.957166	0.971647	0.970588
22.	13	13	26	12	12	4	11	0.985816	0.975963	0.985531	0.985401
23.	13	13	26	12	12	3	11	0.992908	0.989212	0.992822	0.992754
24.	13	13	26	12	12	4	11	0.985816	0.975963	0.985531	0.985401
25.	13	13	26	12	12	4	11	0.985816	0.975963	0.985531	0.985401
26.	15	35	70	14	6	2	3	0.993399	0.99332	0.99332	0.99308
27.	15	35	70	14	6	3	3	0.98	0.968495	0.979163	0.978495
28.	15	21	42	14	10	2	9	0.997856	0.997848	0.997848	0.997824
29.	15	21	42	14	10	2	9	0.997856	0.997848	0.997848	0.997824
30.	15	15	30	14	14	2	13	0.998522	0.998518	0.998518	0.998506
31.	15	15	30	14	14	2	13	0.998522	0.998518	0.998518	0.998506
32.	16	60	120	15	4	2	3	0.994236	0.994171	0.994171	0.993976
33.	16	40	80	15	6	2	5	0.996593	0.996571	0.996571	0.996503
34.	16	40	80	15	6	3	5	0.989717	0.984156	0.989488	0.989305
35.	16	30	60	15	8	4	7	0.985401	0.975079	0.985021	0.984848
36.	16	24	28	15	10	2	9	0.998126	0.998119	0.998119	0.998099
37.	16	24	28	15	10	5	9	0.981176	0.966406	0.980653	0.980488
38.	16	20	40	15	12	2	11	0.99847	0.998465	0.998465	0.998452

39.	16	20	40	15	12	3	11	0.995397	0.993013	0.995352	0.995316
40.	16	20	40	15	12	4	11	0.990783	0.984406	0.990634	0.990566
41.	16	20	40	15	12	6	11	0.977011	0.957992	0.97635	0.97619
42.	16	16	32	15	15	3	14	0.996393	0.994539	0.996366	0.996344
43.	16	16	32	15	15	5	14	0.98797	0.978683	0.987759	0.987692

CONCLUSION:

Mating designs are the study of progenies developed through various methods like Diallel Cross plans which are subjected to Incomplete Block Designs. The analysis of such plans, namely the estimation of variance components, design and genetic, is available in the literature. However, the primary interest in this study is to examine the robustness of various mating designs as it depends on the underlying experimental design. Robustness of Complete Diallel Cross plan is examined using NBIB Design. There are varying Nested Balanced Incomplete Block Designs for different parametric values with unavailability of two blocks. The paper dealt with a class of 43 Nested Balanced Incomplete Block Designs for varying parametric values with unavailability of two blocks and efficiencies are calculated for each case. Efficiencies are varying based on the;

i. number of common lines between two blocks is zero
ii. number of common lines between two blocks is one
iii. number of common lines between two blocks is two or more

Results shows that the efficiencies of case *(ii)*, and case *(iii)* will be more than that of case *(i)*. It appears that Nested Balanced Incomplete Block Designs are fairly robust against the unavailability of two blocks corresponding to the same test treatment.

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